Wind Tunnel Validation of a Particle Tracking Model to Evaluate the Wind-Induced Bias of Precipitation Measurements.

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Abstract

A physical full-scale experimental set-up was designed and implemented in the wind tunnel to reproduce and capture the trajectories of falling water drops when approaching the collector of catching type precipitation gauges, reproducing rainfall measurements in windy conditions. The experiment allowed to collect, for the first time, a large dataset of high-resolution footages of the deviation of such trajectories, as induced by the bluff-body aerodynamics of the outer gauge shape. By processing the collected images, a consistent quantitative interpretation of each drop pattern was possible, based on a detailed Computational Fluid Dynamics simulation of the airflow updraft and acceleration features above the collector of the gauge. Numerical airflow simulations were extensively validated in the wind tunnel, using local flow measurements and Particle Image Velocimetry. Capturing the deviation of the drop trajectories in the wind tunnel allowed a clear visualisation of the physical reason for the wind-induced undercatch of precipitation gauges, since drops were individually observed to fall outside instead of inside of the collector, contrary to what would be expected by extrapolating their undisturbed trajectory. The adopted Lagrangian Particle Tracking model and the formulation used for the drag coefficient were suitable to closely reproduce the observed drop trajectories when affected by the airflow deformation due to the bluff-body aerodynamics of two investigated gauge geometries. The wind tunnel experiment provided the basis for the validation of the particle tracking model in terms of the difference between simulated and observed trajectories, after initial conditions were suitably set to represent the experimental setup.

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9 Key Points:

- Water drops are released in a wind tunnel to mimic rainfall and tracked to observe the
 wind-induced measurement bias of raingauges
- Numerical simulation of the airflow field and a lagrangian particle tracking model are
 applied to reproduce the drop trajectories
- Wind tunnel tests validate airflow simulation and particle tracking results supporting their
 application in studying the wind-induced bias
- 16

18 Abstract

19 A physical full-scale experimental set-up was designed and implemented in the wind tunnel to reproduce and capture the trajectories of falling water drops when approaching the 20 collector of catching type precipitation gauges, reproducing rainfall measurements in windy 21 conditions. The experiment allowed to collect, for the first time, a large dataset of high-22 23 resolution footages of the deviation of such trajectories, as induced by the bluff-body aerodynamics of the outer gauge shape. By processing the collected images, a consistent 24 quantitative interpretation of each drop pattern was possible, based on a detailed Computational 25 Fluid Dynamics simulation of the airflow updraft and acceleration features above the collector of 26 the gauge. Numerical airflow simulations were extensively validated in the wind tunnel, using 27 local flow measurements and Particle Image Velocimetry. Capturing the deviation of the drop 28 29 trajectories in the wind tunnel allowed a clear visualisation of the physical reason for the windinduced undercatch of precipitation gauges, since drops were individually observed to fall 30 outside instead of inside of the collector, contrary to what would be expected by extrapolating 31 their undisturbed trajectory. The adopted Lagrangian Particle Tracking model and the 32 formulation used for the drag coefficient were suitable to closely reproduce the observed drop 33 trajectories when affected by the airflow deformation due to the bluff-body aerodynamics of two 34 investigated gauge geometries. The wind tunnel experiment provided the basis for the validation 35 of the particle tracking model in terms of the difference between simulated and observed 36 trajectories, after initial conditions were suitably set to represent the experimental setup. 37

38 **1 Introduction**

The external design of catching type precipitation gauges has a major impact on the catch efficiency in windy conditions due to the bluff-body aerodynamics of their outer geometry. This topic is numerically investigated by using Computational Fluid Dynamics (CFD) simulations of the airflow field surrounding the gauge body (see e.g. Colli et al., 2018) and by adopting a Lagrangian Particle Tracking (LPT) model to evaluate the airflow induced deformation on the trajectory of the approaching hydrometeors (see e.g. Nešpor & Sevruk, 1999).

The work of Mueller & Kidder (1972) is among the earliest studies on the modelling of 45 hydrometeor trajectories using a LPT model; in that work, particle trajectories were numerically 46 simulated based on the flow field measured in the wind tunnel (WT) employing hot film 47 anemometers. Folland (1988) later developed two simplified trajectory models to estimate the 48 catch losses due to the wind. They were based on flow patterns (flow velocity and direction) 49 obtained in the vertical section along the symmetry axis in the streamwise direction partially 50 published by Robinson & Rodda (1969), and the flow field close to the windward edge as 51 described in the work of Warnik (1953). In the first model, a two-dimensional solution was 52 obtained by simulating one by one droplets of fixed diameter until the expected drop size 53 distribution was generated. Then the model was extended to a three-dimensional domain by 54 considering the variation of the gauge orifice width in the transversal direction. The second 55 model, which the author terms "semi-analytical", uses geometrical considerations to calculate the 56 number of particles which fall outside instead of inside of the gauge collector. 57

In the work of Nešpor & Sevruk (1999), the airflow velocity field around three cylindrical gauges characterized by different shapes of the collector rim was calculated by numerically solving the Reynolds Average Navier-Stokes (RANS) equations based on the k- ϵ turbulence closure model. Then, liquid particle trajectories were modelled by employing a one-

way coupled model where spherical particles are separately simulated for each diameter and the 62 Collection Efficiency (CE) is evaluated by computing the integral, over the particle size 63 distribution, of the number of particles collected by the gauge with respect to the total 64 precipitation. This simulation scheme was also adopted by Thériault et al. (2012) and Colli et al. 65 (2015, 2016 a,b) for solid precipitation, by increasing the details of the computational mesh to 66 better capture the airflow features and using also Large Eddy Simulations (LES). In the work of 67 Thériault et al. (2012) different crystal types were modelled by using a power law 68 parametrization of the terminal velocity, volume, density and cross section of the particles, while 69 Colli et al. (2016b) investigated two macro categories: wet and dry snow as suggested by 70 Rasmussen et al. (1999). In these works, a fixed drag coefficient for each crystal type was 71 adopted. In the work of Colli et al. (2016b) the obtained collection efficiency curves for the two 72 macro categories act as upper and lower thresholds of the wide spreading of experimental data. 73 Colli et al. (2015) obtained a better comparison with real-world data by introducing the 74 dependence of the aerodynamic Drag Coefficient (C_D) on the local Reynolds number of the 75 simulated particle (Re_p) . The Re_p is function of the instantaneous particle-to-air magnitude of 76 velocity and the particle trajectory is obtained by updating its value and the associated C_D at 77 each simulation time step. 78

Due to the difficulties in reproducing hydrometeors trajectories under controlled WT conditions, or to observe their deflection when approaching the gauge in any real world configuration, the validation of trajectory models is scarcely documented yet in the literature. Very few literature works, Warnik (1953) and Green & Helliwell (1972), report some attempts to detect water drops in WT.

The WT experiments performed by Warnik (1953) were a pioneer work where the 84 trajectories of precipitating solid particles under windy conditions and their deviations when 85 approaching the gauge were first visualized. However, no attempt to quantitatively measure and 86 model the deviation of particles in the presence of the gauge collector was made and just the 87 overall undercatch was quantitatively obtained. The main objective was to define the air-flow 88 behavior around the investigated gauges and to give definite information about the 89 inconsistencies observed in records of snow catch in precipitation gauges. Results were used to 90 design the geometry of two windshields with better performance than the shields in use at that 91 92 time. Green & Helliwell (1972) measured wind velocity profiles and raindrop trajectories above a cylindrical rain gauge in a WT. The flow field was measured using a grid of hotwire 93 94 anemometers and the trajectories of injected drops using photographs from a still reflex camera. Furthermore, they simulated the drop trajectories based on the streamlines obtained by visually 95 interpreting the pattern of smoke injected in the WT. Little details are provided about the 96 algorithms and procedures used in that work. 97

The original contribution of the present work consists of a dedicated WT experimental setup, where a physical full-scale model was designed and realized in order to release water drops in the WT flow field and to detect – at high resolution – their deviated trajectories close to the gauge collector, where the airflow field is modified by the presence of the gauge body.

Dedicated numerical simulation of the experimental setup was performed by adapting the CFD approach proposed by Nešpor & Sevruk (1999), Thériault et al. (2012) and Colli et al. (2016 a) and implementing the LPT model with suitable initial conditions (drop velocity and position), as observed in the WT. The improved LPT model used by Colli et al. (2015) for solid precipitation was adapted to simulate the trajectories of water droplets when falling through the 107 atmosphere and approaching the gauge collector using suitable drag coefficient equations as a 108 function of the local Reynolds number of the simulated particles. Comparison between the 109 simulated and observed trajectories of droplets of various size and fall height allowed full 110 validation of the LPT model in a controlled WT environment.

111 2 Methodology

The most popular outer shapes of catching type precipitation gauges, cylindrical and chimney-shaped (or conical), were investigated. Although actually exploiting a weighing measurement principle, the *Lambrecht Rain[e]H3*[©] precipitation gauge has a cylindrical outer shape (see Figure 1 top-left) and is assumed here as representative of the majority of tippingbucket gauges, while the *Geonor T200B*[©] has a chimney shape (see Figure 1 bottom-left) that is typical of most weighing type gauges.

Within the *PRIN 20154WX5NA* project, an extensive experimental campaign was conducted in the WT facility available at Politecnico di Milano, hereafter GVPM, and in the WT of the DICCA department at the University of Genova. This included Particle Image Velocimetry (PIV) and local flow velocity measurements, and the release and tracking of droplets using a drop generator and a high-speed camera.

123 2.1 Wind Tunnel experiments

The numerical velocity fields were validated by comparison with PIV measurements along two vertical sections in the 2-D (x, z) plane, where x is the stream-wise direction, above the *Geonor T200B*[©] collector and with local flow velocity measurements obtained in selected positions close to the *Lambrecht Rain[e]H3*[©] collector.

For PIV measurements, the GVPM test chamber was uniformly filled with castor oil 128 smoke, used as a tracer. A laser emitter was mounted on the ceiling of the test chamber to 129 illuminate the measurement plane, while the surrounding environment was kept in the dark. The 130 video camera was positioned with its central axis normal to the stream-wise direction and 131 centered on the transversal symmetry axis of the collector of the Geonor $T200B^{\odot}$. Post 132 processing of the acquired images provided the flow velocity field discretized in a regular grid 133 134 with cell size of 7.5 x 7.5 mm. Part of the data, acquired very close to the gauge surface and 135 disturbed by the reflection of the laser beam on the gauge rim, were masked out.

The Lambrecht Rain[e]H3[©] precipitation gauge was tested in the DICCA WT. Local measurements of the wind speed were acquired using a fast-response multi-hole probe, the "Cobra" probe, characterized by an measuring cone of $\pm 45^{\circ}$, mounted on a 3 degrees of freedom traversing system. Each measurement was sampled at 2 kHz for 30 seconds. The measurement positions were selected above the gauge along two longitudinal lines at different elevations and along two vertical lines at the center and the upwind edge of the collector.

With the aim to validate the LPT model, water drops were injected in the GVPM WT and their trajectories were captured with a high-speed camera in the vertical 2-D plane along the main flow direction. The deviation of the drop trajectories approaching and travelling above the gauge collector was measured. The WT was equipped with a hydraulic system to generate water droplets, a high-speed camera and a high-power lamp to photograph and illuminate the droplets along their trajectories. The experimental setup is illustrated in the right panel of Figure 1. The hydraulic system consisted of a tank with a constant water head, feeding a volumetric pump (Ismatec Reglo-Z digital) connected to a calibrated nozzle; the droplets releasing frequency could be adjusted by varying the pump flow rate between 0.4 and 18.0 ml/min. Tests were conducted using a fixed flow rate of 0.8 ml/min and a nozzle orifice size of 0.008". The support for the nozzle was shaped to have a reduced impact on the airflow close to the orifice and to minimize the oscillations at the edge where droplets are released.



Figure 1. Lambrecht Rain[e]H3© (top-left) and Geonor T200B© (bottom-left) precipitation gauges and experimental setup to release drops and photograph their trajectories installed in the GVPM.

The high-speed camera was placed in front of the gauge and its distance from the gauge 158 collector was optimized to increase the resolution of the captured drop images without affecting 159 the airflow field near the target. The focus plane was set as the vertical section along the 160 streamwise symmetry axis of the gauge collector. Acquisition of video sequences was carried out 161 at both a high-frame rate (fps) and a low-frame rate. In the first case, the recording speed was 162 optimized to maintain high quality of the images, in terms of resolution and luminosity, suitable 163 for capturing the drop movement with no streaks appearing due to an excessive exposure time 164 with respect to the drop displacement. The recording speed was set to 1000 fps, with an image 165 resolution of 1600x900 pixels. During the low frame rate acquisition, the recording speed was 166 kept low, 10 fps with the same image resolution, in order to increase the exposure time. In this 167 way the trajectory of a bright moving object was imprinted in a single image in the form of a 168 169 single streak.

The GVPM WT campaign was realized in subsequent time slots, in each of them the 170 dimension of the image was about 40 x 20 cm with the pixel size equal to 0.2370 mm for the 171 GeonorT200B[°] gauge and 0.2424 mm for the for Lambrecht Rain[e]H3[°]. The wind speed range 172 investigated was between 9 and 13 m s⁻¹. The lighting was designed to increase the drops 173 visibility in the videos: an incandescent lamp was used to illuminate the volume near the gauge 174 175 collector. Tests were conducted in a dark environment while backlighting the gauge from above using the lamp. The lamp lights are suitably directed to avoid the saturation of the recorded 176 videos. 177

The investigated precipitation gauges were positioned downstream of the drop generator, so that drops were released along the streamwise symmetry axis of the gauge. The drop release position was determined a priori using the LPT model based on the flow field provided by CFD

simulations at a wind speed equal to 10 m s⁻¹. The drop size was set equal to the minimum 181 182 dimension that the video camera is able to capture (1 mm) following dedicated tests performed in still air. The initial velocity of the drop was imposed equal to zero in all directions. This 183 configuration allowed a significant deformation of the trajectories to be observed when drops 184 travelled in the region where the flow field is disturbed by the presence of the gauge. This 185 configuration was chosen since drops could not be generated with their terminal velocity, 186 otherwise – given the above mentioned constraints on the drop size – the high inertia of such 187 large drops would have resulted into an imperceptible deflection of the drop trajectories above 188 the gauge collector. 189

The videos recorded by the camera were imported and analyzed in the MATLAB® 190 environment. The methodology of detection of the drop position differs depending on the type of 191 acquisition. In high-fps, the path of each drop is identified by many frames: in every frame the 192 drop is in a different position. Due to the reduced exposure time of each image, little light enters 193 into the camera sensor and the image is dark. To improve the drop position identification, each 194 image was then converted to greyscale and a combination of a Gaussian and Laplacian filter was 195 applied. The image was binarized using a threshold level, with the zeroes indicating the 196 background color while the ones indicate the drop position. Finally, using a moving window over 197 the image, the center of the drop was identified and stored. Knowing the time interval between 198 199 two subsequent images and the conversion rate from pixels to mm, it was also possible to calculate the drop speed in the 2-D shooting plane. In low-fps acquisition, due to the high 200 exposure time, every frame appears much more bright and contains a streak representing the 201 trajectory of each single drop. The same filtering and binarization operations were used, adapting 202 the filter and threshold parameters. Finally, morphological operations were carried out, in order 203 to extract from the image the middle line streak, directly corresponding to the trajectory of the 204 drop. 205

206 2.2 CFD simulations

To obtain the modified airflow velocity fields (magnitude and components) around the 207 two investigated gauges, CFD simulations were performed under constant wind velocity and 208 uniform incoming free-stream conditions. First, an Unsteady Reynolds Average Navier-Stokes 209 (URANS) model was applied to the simulation of the flow field around the Geonor $T200B^{\odot}$ 210 gauge geometry in order to check the occurrence of any variation in time of the average velocity 211 field. Results revealed that a steady state solution is reached in the region of major interest for 212 this work, i.e. above the gauge collector, and only beyond the gauge, in the wake, a time 213 dependent solution persists. For this reason, in order to reduce the computational burden, a 214 RANS model was used in the simulation of the Lambrecht Rain/e/ $H3^{\odot}$ gauge. In both cases, the 215 SST (Shear Stress Transport) k-ω turbulence model, based on the (specific) turbulence kinetic 216 217 energy (k), the energy dissipation per unit mass (ε) and the turbulent specific dissipation rate (ω) as defined in Wilcox (2006), was adopted. The SST k-w closure model was developed by Menter 218 (1992) and concentrates the advantages of the more classic k- ε and k- ω models because it is able 219 to switch to a k- ε behaviour in the free stream far from the object and to the k- ω model near the 220 walls. Indeed, the k- ε model formulated by Jones & Launder, (1973), is robust and reliable in the 221 free flow region but it proved to be unreliable near the boundary layer, while the k- ω model 222 223 formulated by Wilcox (1988) is capable of correctly modelling turbulence near the boundary layer, though presenting a strong dependency on arbitrary values in the free flow. Concerning the 224 topic of this work, Constantinescu et al. (2007) tested different numerical methods to investigate 225

the shielding problem between two contiguous precipitation gauges, and concluded that the SST $k-\omega$ model is more consistent with the time-dependent LES results on the upstream gauge, in conditions that are similar to the present work.

Although it was demonstrated (Cauteruccio et al., 2020) that the free-stream turbulence intensity inherent to the natural wind has a significant role in attenuating the aerodynamic effect of precipitation gauges, due to the energy dissipation induced by turbulent fluctuations, in this work a low free-stream turbulence condition was used to reproduce the WT environment where the particle tracking experiment was carried out.

To prepare the simulation setup, first the numerical models of the gauge geometries were 234 realized in the Standard Triangulation Language (STL) format using a 3D CAD software. The 235 three-dimensional computational spatial domains were discretized using an unstructured hybrid 236 hexahedral/prismatic finite volume mesh. The number of cells of the computational mesh is 1.5 x 237 10^6 for the Geonor T200B[©] and doubled for the Lambrecht Rain[e]H3[©] due to the sharp edge 238 that characterized the rim geometry, which requires an accurate discretization. In both cases, 239 mesh refinement boxes and thin layers were realized close to the gauge and on its surface in 240 order to increase the accuracy of the numerical solution in the region affected by large gradients 241 of velocity and pressure. The quality of the mesh was checked by using the geometry parameters 242 of orthogonality, skewness and aspect ratio. 243

The open-source OpenFOAM software was employed to solve the URANS and RANS 244 equations. The fluid air was modelled as a Newtonian incompressible fluid with kinematic 245 viscosity $v_a = 1.5 \times 10^{-5} \text{ m}^2 \text{s}^{-1}$ and density $\rho_a = 1.25 \text{ kg m}^{-3}$ at a reference environmental temperature $T_a = 20 \text{ °C}$. For each configuration, at the inlet of the computational domain (y-z 246 247 plane) the undisturbed wind speed, U_{ref} , equal to 10 ms⁻¹ was imposed parallel to the x axis and it 248 was maintained uniform and constant in time, while a null gradient condition was set for 249 pressure. At the outlet, y-z plane opposite to the inlet, the atmospheric pressure and a null 250 gradient condition for the velocity were imposed. The lateral surfaces of the domain were set as 251 symmetry planes, while the ground and the gauge surface were assumed impermeable with a no-252 slip condition. In all computational cells, initial conditions were imposed equal to U_{ref} for the 253 254 velocity and equal to zero for the relative pressure.

255 2.3 The Lagrangian Particle Tracking model

The LPT model used by Colli et al. (2015) for solid precipitation was modified by 256 257 introducing drag coefficient equations suitable for liquid precipitation. Drop trajectories were computed with a forward step procedure by calculating at short time intervals the particle 258 position, velocity and acceleration. The relative particle-to-air velocity was updated at every time 259 step by interpolating the CFD airflow field to obtain the flow velocity in the exact position of the 260 drop. This model is one-way coupled since the potential influence of the particles on the airflow 261 field is neglected. This simplification is acceptable since the particles concentration in the air is 262 very low as observed by Cauteruccio (2020). A spherical shape was assumed for the water drops, 263 with the associated equivalent diameter d, and the particle density was set equal to 1000 kg m⁻³ at 264 the air temperature of 20°C. The drag coefficient equations were implemented for various ranges 265 of particle Reynolds number established a priori among those proposed in the literature by 266 Folland (1988) and formulated starting from data published by Beard (1976) and Khvorostyanov 267 and Curry (2005), hereinafter KC05. 268

269

The motion of particles when falling in the atmosphere is described by:

$$\rho_p V_p \boldsymbol{a}_p = -\frac{1}{2} C_D A_p \rho_a (\mathbf{v}_p - \mathbf{v}_a) |\mathbf{v}_p - \mathbf{v}_a| + V_p (\rho_a - \rho_p) \boldsymbol{g}$$
(1)

where \mathbf{a}_p is the particle acceleration, \mathbf{v}_a and \mathbf{v}_p are the velocity vectors of the air and the particle, \mathbf{g} is the gravity acceleration, C_D is the drag coefficient, A_p is the particle cross section area and ρ_a and ρ_p are the density of the air and the particle. Equation 1 assumes an upward positive orientation of the z axis, while the velocity and acceleration components are positive in the positive direction of the related axes. The quantity $\mathbf{v}_p - \mathbf{v}_a$ is the relative particle-to-air velocity.

The drag coefficient (C_D) is a dimensionless quantity used to represent the aerodynamic 276 resistance of an object in motion in a fluid, such as air or water, and depends on the cross-277 sectional area of the object (A_p) . The estimation of the drag coefficient is not easy. In the 278 literature, different experiments were carried out with hydrometeors falling through the 279 280 atmosphere with the objective of identifying suitable relationships between the drag coefficient and the particle dimension and/or its terminal velocity. Beard (1976) derived the C_D by 281 performing experiments in stagnant air, and measuring the dimension d [mm] of the drops and 282 their fall velocity w_T [m s⁻¹]. 283

284

When a particle falls in a stagnant air its Reynolds number is expressed as:

$$Re_p = \frac{w_T d}{v_a} \tag{2}$$

where v_a is the air viscosity in m² s⁻¹. The drag coefficient is directly related to the Reynolds number of the particle in motion. When a particle is immersed in a flow field the equation 2 becomes a function of the particle-to-air velocity as follows:

$$Re_p = \frac{|\mathbf{v}_p - \mathbf{v}_a|d}{v_a} \tag{3}$$

Folland (1988) proposed different relationships between Re_p and C_D , for various ranges of Re_p (see equations 4 to 7) and assumed that the minimum value for C_D must be fixed at 0.55.

291

$$Re_p < 0.01$$
 $C_D = 2547$ (4)

292

$$0.01 \le Re_p < 2 \qquad C_D = 1.06(24 Re_p^{-1} + 2.400 Re_p^{-0.045}) \tag{5}$$

293

$$2 \leq Re_p \leq 21 \qquad C_D = 1.06(24 Re_p^{-1} + 2.640 Re_p^{-0.19}) \tag{6}$$

294

$$21 < Re_p \qquad C_D = 1.06 \left(24 Re_p^{-1} + 4.536 Re_p^{-0.368} \right) \tag{7}$$

The equations proposed by Folland (1988) provide terminal velocities of raindrops in still air at the temperature of 7.5°C and atmospheric pressure that agree to within 2% with those of Mason (1971) over the range of terminal velocities $0.1 < w_T < 8.3$ m s⁻¹.

In the work of KC05, different formulations of C_D , as a function of Re_p obtained from experimental studies and analytical models, were summarized. Some of the curves proposed by KC05 are derived for spherical particles and other for crystals. The authors also introduce a correction, when the particle Reynolds number exceeds the value of 10^3 , in order to account for the turbulence effect due to the flow.

The C_D values proposed by KC05 as a function of Re_p , for spherical particles in turbulent conditions, were fitted here for $Re_p > 60$ (approximately corresponding to a spherical drop of d= 1 mm falling in air, with $v_a = 1.5 \ 10^{-5} \ m^2 s^{-1}$, at an indicative fall velocity of 1 m s⁻¹) with an inverse first-order equation (Eq. 8). The values of the three parameters are $y_0 = 0.442$, a =3.402, b = 21.383 and the correlation factor (R²) is equal to 0.996.

$$C_D = y_0 + \frac{(a b)}{b + Re_p} \tag{8}$$

As shown in Figure 2, also the relationships proposed by Folland (1988) (black line) were compared with the raw data provided by KC05 (diamond). Based on this comparison the best-fit curve reported in Eq. 8 (grey line in Figure 2) was implemented in the LPT model to calculate the drag coefficient for $Re_p > 400$, while the equations proposed by Folland (1988) were adopted for $Re_p < 400$.





Equation 8 (grey line) and the formulation proposed by Folland (1988) (black line).

317 **3 Results and discussion**

318 3.1 CFD simulation results and validation

The performed CFD simulations confirmed previous literature results (e.g. Colli et al., 319 2018 and Cauteruccio, 2020): for both gauge geometries the airflow accelerates above the 320 collector and significant updraft is obtained just in front of the gauge collector and above it. The 321 region of maximum acceleration for the *Geonor T200B[©]* gauge shows higher values and is larger 322 and more dispersed upward if compared with the aerodynamic response of the Lambrecht 323 *Rain[e]H3* gauge. The maximum normalized updraft U_{τ}/U_{ref} occurs above the upwind edge of 324 the collector and, e.g. at z/D = 0.125, equals 0.65 and 0.50 for the Geonor $T200B^{\odot}$ and the 325 Lambrecht Rain/e/H3 gauge, respectively. These airflow features are the most influential on the 326 hydrometeor trajectories, which are deflected upward due to the updraft and tend to be dragged 327 beyond the collector when falling through the accelerated zone, where the horizontal velocity is 328 329 larger than the undisturbed wind speed.



Figure 3. Comparison between URANS velocity profiles (continuous lines) and PIV measurements (markers) of the normalized magnitude of flow velocity (U_{mag}/U_{ref}) at $U_{ref} = 10$ m/s and at different elevations along the central plane (y/D = 0) of the *Geonor T200B*[©] gauge (top panel) and their absolute difference ΔU^* (bottom panel).



Figure 4. Comparison between URANS velocity profiles (continuous lines) and PIV measurements (markers) of the normalized magnitude of flow velocity (U_{mag}/U_{ref}) at $U_{ref} = 10$ m/s and at different elevations along the plane y/D = 0.25 of the *Geonor T200B*[©] gauge (top panel) and their absolute difference ΔU^* (bottom panel).





Figure 5. Comparison between URANS velocity profiles (continuous lines) and *Cobra* probe velocity measurements (circles with uncertainty bars) of the normalized magnitude of flow velocity (U_{mag}/U_{ref}) at $U_{ref} = 10$ m/s in the central plane (y/D = 0) along two longitudinal sections at different elevations z/D (left-hand panel) and along two vertical sections in the upwind edge (x/D = -0.5) and in the center (x/D = 0) of the collector (right-hand panel) for the *Lambrecht Rain[e]H3*[©] gauge.

Validation of the performed simulations was obtained in the WT using PIV and Cobra 347 probe measurements. The comparison between PIV measurements and CFD longitudinal profiles 348 of the normalized magnitude of the flow velocity at $U_{ref} = 10 \text{ m s}^{-1}$ for the Geonor $T200B^{\odot}$ 349 gauge are reported in Figure 3 and Figure 4. The plots refer to different elevations (z/D) and 350 vertical sections, at y/D = 0 and y/D = 0.25, respectively, and the absolute difference (ΔU^*) 351 between the measured and calculated values is also reported. An overall good agreement of the 352 simulated profiles with measurements is obtained. The maximum differences occur at the 353 minimum elevation where PIV measurements are available (z/D = 0.23) and range between 0.2 354 and 0.3. This is due the coarse discretization of the measured spatial domain, which introduces a 355 smoothing effect on the measured flow field. Moreover, close to the gauge, measurements are 356 still slightly affected by the reflection of the laser beam on the gauge rim, partially concealing 357 the passive tracer from detection despite a mask was implemented to reduce such effect. 358

For the *Lambrecht Rain[e]H3*[©] four velocity profiles were sampled above the collector, along the longitudinal symmetry plane (y/D = 0) of the gauge, by employing a *Cobra* probe. The comparison between the local flow velocity measurements (circles) and numerical results (solid lines) are shown in Figure 5. The longitudinal profiles were measured at z/D = 0.125 and z/D =0.25, while the vertical profiles at the center and the upwind edge of the collector (x/D = -0.5 and x/D = 0). In all cases, the numerical profiles fall within the uncertainty of the probe measurements, depicted with bars, confirming the good quality of the adopted CFD setup.

366 3.2 Observation of the deflected trajectories

For the two investigated gauges, a few sets of drops captured in the GVPM WT when 367 travelling above the gauge collector are shown in this section, and the observed trajectories are 368 commented according to the aerodynamic response of the gauge body. Drop trajectories are 369 depicted in the dimensionless plane (x/D, z/D), where the gauge collector is centered in (0, 0) and 370 the longitudinal coordinate, x/D, of the drop releasing position is fixed, while its elevation, z/D371 assumed positive downward, varies with the wind speed. In the legend, the prefix in the coding 372 of each trajectory indicates the name of the gauge under test (G for GeonorT200B[©] and L for 373 Lambrecht Rain $[e]H3^{\odot}$). 374

Overall, the experimental campaign in the WT resulted in the capturing of 82 trajectories of drops released between z/D = -0.05 and z/D = -0.7 above the *GeonorT200B*[®], at $U_{ref} = 8.86$ to 13.6 m s⁻¹, and 106 trajectories of drops released between z/D = -0.18 and z/D = -0.7 above the *Lambrecht Rain[e]H3*[®], at $U_{ref} = 8.9$ to 13.1 m s⁻¹.

Two sample sets of drop trajectories shot at 1000 and 10 fps are shown in Figure 6 (left 379 and right panel, respectively), as observed above the collector of the cylindrical and chimney 380 shaped gauges at $U_{ref} = 12.5 \text{ m s}^{-1}$ and $U_{ref} = 11.4 \text{ m s}^{-1}$, respectively. The particle-fluid 381 interaction above the collector of the gauges is responsible for a significant deviation of the 382 trajectories, and this can be observed here for all trajectories when travelling beyond the upwind 383 384 edge of the collector (x/D = -0.5). In both cases, a few undisturbed trajectories aiming at entering the collector close to the downwind edge (x/D = 0.5) overtake its rim and fall outside of the 385 gauge. The shooting of the deviated trajectories in the WT allowed clear visualization of the 386 wind-induced undercatch by showing that some drops actually fall outside instead of inside of 387 388 the collector, contrary to what would be expected by following their undisturbed trajectory.





The estimated undisturbed trajectory is added in the top panel of Figure 7 (dashed line) for one sample drop travelling above the *Lambrecht Rain[e]H3[©]* gauge at $U_{ref} = 12.5 \text{ m s}^{-1}$. In this case, the observed trajectory starts detaching from the undisturbed one when its longitudinal coordinate reaches the upwind edge of the collector (at about x/D = -0.5), where the airflow updraft is most significant (as reported by Cauteruccio, 2020).

The observed drop trajectory was elaborated by linearly interpolating the positions 398 associated with the undisturbed part of the trajectory, while the disturbed part was fitted with a 399 third order polynomial. In Figure 7 (top panel), the undisturbed part of the observed trajectory is 400 painted with a dark grey while the disturbed part is painted in light grey and the interpolation 401 curves are marked with dots. The threshold between the undisturbed and disturbed part of the 402 trajectory was obtained by adopting a trial and error procedure with the objective to ensure the 403 continuity of the slope curve (z/x) obtained as the first derivative of the fitted trajectory (Figure 404 7, bottom panel). 405



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Figure 7. Observed (circles) and undisturbed (dashed line) drop trajectory above the collector of the gauge with cylindrical shape (top panel) at $U_{ref} = 12.5 \text{ m s}^{-1}$, and the associated slope curves for both trajectories (bottom panel).

Two drop trajectories, identified with the names G8 and G9, travelling above the collector of the *Geonor T200B*[©] gauge at $U_{ref} = 10.2 \text{ m s}^{-1}$ are depicted in Figure 8. The initial elevation (*z/D*) of the two droplets is not much different. The initial, undisturbed part of the slope curve reveals that the two droplets have a similar size, while the drop starting at a higher elevation is deviated a bit later following the airflow pattern. The difference between the observed and undisturbed trajectories, red markers with scale on the right-hand axis, is larger in the second half part of the collector where the two droplets are dragged beyond the gauge, resulting in some undercatch.



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Figure 8. Comparison of two drop trajectories (circles) having similar size, travelling above the collector of the chimney shaped gauge at $U_{ref} = 10.2 \text{ m s}^{-1}$, together with the associated undisturbed trajectories (dashed lines). The associated slope curves (scale on the right-hand axis) are shown with colored solid and dashed lines for the disturbed and undisturbed trajectories, respectively. Red markers represent the difference between the observed and undisturbed trajectories in terms of normalized vertical coordinates (scale on the right-hand axis).

425 3.3 Validation of the Lagrangian Particle Tracking model

The validation of the LPT model was obtained by comparison between observed and 426 simulated trajectories. In the numerical model the initial conditions, normalized position (x/D)427 and z/D) and velocity components (u and w [m s⁻¹], the latter assumed positive downward), of the 428 simulated trajectories were set consistently with the WT observations. CFD airflow velocity 429 fields were rescaled according to the free-stream velocity value used in the WT. The initial 430 velocity components were set equal to the mean values of the three to five initial positions of 431 each drop as shot by the camera, so as to avoid the noise due to the uncertainty in the initial 432 positions. The drop diameter, d, which has a major role in the calculation of the drop trajectory 433 by affecting Re_n and therefore C_n , is obtained here as a calibration parameter for each trajectory, 434 since the drop releasing mechanism and the acquisition system cannot provide sufficient 435 accuracy in the assessment of the drop size. The initial conditions of the drop trajectories used in 436 the validation phase and their estimated diameter are listed in Table 1. A few examples of 437 simulated drop trajectories, compared with the observed ones, are presented and discussed in this 438 439 section to support the validation phase.

The observed (circles) and simulated (solid line) patterns of the drop trajectory identified 440 with the name G2, are compared in Figure 9. The maximum difference between the vertical 441 positions (z in mm), computed at each normalized longitudinal coordinate (x/D) of the observed 442 trajectory, arises at the upwind edge of the collector and is about 1.2 mm. This difference is 443 comparable with the drop size (see Table 1), therefore with the uncertainty in the assessment of 444 the drop position, identified as a bright moving object in each frame. The calculated horizontal 445 acceleration of the drop, normalized with the one experienced in the initial undisturbed part of 446 the trajectory, is also shown (scale on the right-hand axis). Consistently with the PIV airflow 447

velocity fields, the drop significantly accelerates when travelling above the upwind part of the collector, where the airflow is indeed accelerated, until crossing the separation layer between the airflow recirculation and accelerated zones, when it starts decelerating abruptly towards the downwind edge of the collector.



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Figure 9. Observed (circles) and simulated (solid line) drop trajectories above the collector of a chimney shaped gauge at $U_{ref} = 10.2 \text{ m s}^{-1}$. The difference dz (mm) between the observed and simulated vertical coordinates of the drop trajectories (red crosses) at each normalized observed longitudinal coordinate (x/D) is reported (scale on the right-hand axis), together with the numerical longitudinal acceleration (a_x) of the drop (dashed line).

The good repeatability of the trajectories of very similar drops in the WT (see their estimated size in Table 1) is shown in Figure 10. By injecting drops of the same size in the WT, the observed trajectories are indeed very close to each other, and they experience very similar deviations above the collector. Moreover, simulated trajectories show that the LPT model is able to replicate even the small variations due to slight differences in the initial conditions about the drop velocity.



Figure 10. Pairs of drops having approximately the same size travelling above the collector of a chimney shaped gauge at $U_{ref} = 10.2 \text{ m s}^{-1}$ (left-hand panel) and a cylindrical gauge at $U_{ref} = 13.1 \text{ m s}^{-1}$ (right-hand panel). Observed (circles) and simulated (lines) trajectories are depicted for the two pairs of drops.

The observed drop trajectories G8 and G9, already shown in Figure 8, are compared with 469 470 the simulated trajectories in **Figure 11**. As already noted, the two droplets have the same size; this assumption is confirmed by the obtained numerical trajectories because the optimal 471 agreement between the observed and simulated trajectories is reached by setting the drop 472 diameter equal to one millimeter. Also in this case, the difference between observed and 473 simulated trajectories is comparable with the uncertainty in the assessment of the drop position, 474 given the dimension of the drop (see Table 1) and the image resolution. The vertical acceleration 475 of the drops (a_7) obtained from the simulation, normalized with the one experienced in the initial 476 undisturbed part of the trajectory, is depicted with dashed lines: the two drops accelerate when 477 reaching the upwind edge of the collector, due to the updraft, and then decelerate. This behavior 478 is in line with the measured PIV velocity field where, beyond x/D = -0.25, the updraft zone is 479 always located above the normalized elevation |z/D| = 0.2 and the two drops are fully immersed 480 in the recirculation zone. 481



Figure 11. Comparison of two simulated and observed trajectories for two drops travelling above the collector of the chimney shaped *Geonor*[©] gauge at $U_{ref} = 10.2 \text{ m s}^{-1}$, together with the simulated profiles of their normalized vertical acceleration (scale on the right-hand axis).



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Figure 12. Observed (circles) and simulated (lines) drop trajectories for two drops released at the same initial position travelling above the collector of the cylindrical gauge at $U_{ref} = 13.1 \text{ m s}^{-1}$.

Observed and simulated patterns are depicted in **Figure 12** for a pair of drop trajectories (named L97 and L100) having different drop size (d = 0.7 and 0.9) but being released at the same initial position. As reported in Table 1, also the values of the initial velocity components are very similar. The lighter drop has a lower slope and is maintained at a higher elevation along the entire trajectory. As revealed by the PIV velocity field, this is due to the persistence of updraft velocity components above |z/D| = 0.25 within the limits of the collector, while below this vertical coordinate the separation layer occurs and downward vertical velocity components arise.

Table 1. WT flow velocity, initial coordinates and velocity components for the simulated drop trajectories and the resulting drop diameter.

ID	Wind $[m s^{-1}]$	x/D	z/D	u [m s ⁻¹]	w [m s ⁻¹]	<i>d</i> [mm]
G2	10.2	-1.259	-0.690	4.286	0.952	1.2
G4	10.2	-1.257	-0.523	4.140	1.104	1.0
G5	10.2	-1.257	-0.523	4.122	1.122	1.0
G8	10.2	-1.247	-0.455	4.048	1.190	1.0
G9	10.2	-1.256	-0.477	3.968	1.190	1.0
L84	13.1	-1.280	-0.349	5.571	0.667	0.7
L86	13.1	-1.293	-0.343	5.801	0.602	0.7
L97	13.1	-1.392	-0.389	5.855	0.482	0.7
L100	13.1	-1.379	-0.393	5.667	0.524	0.9

Validation of the coupled CFD and LPT approach was obtained after numerically 499 500 simulating about 25 trajectories of drops released in the WT experiment. A synthesis of the validation performance is reported in Table 2 The maximum, mean and median difference, dz, 501 502 between the vertical coordinates along each observed and simulated trajectory, normalized with the estimated drop diameter, d, were calculated as suitable performance parameters. For each of 503 them, the maximum, minimum, mean and standard deviation values obtained over the set of all 504 trajectories used in the validation exercise are listed in Table 2. The same statistics are included 505 for the root mean square difference between the observed and simulated vertical coordinates 506 along each trajectory. 507

The validation was satisfactory since the mean of the maximum differences between the simulation and the observed trajectory is about unity, with a low standard deviation (the coefficient of variation is about 0.4). Also, the mean and median values of dz are very similar, showing a symmetrical spread of these differences around the perfect agreement, which suggests quite a random nature of the error. Finally, the RMSD is very low, always below 10^{-3} , indicating an overall good agreement for all pairs of simulated and observed trajectories.

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515	Table 2.	Validation	statistics	for the	selected	performance	parameters
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	Max (dz/d)	Mean (dz/d)	Median (dz/d)	RMSD
Max	2.1492	1.0444	1.2154	0.000812
Min	0.1871	-0.4230	-0.4946	0.000213
Mean	1.0968	0.0332	0.0337	0.000429
Std dev	0.4351	0.3514	0.3905	0.000168

516 4 Conclusions

Although the experimental conditions here implemented in the WT were forcedly different from reality (constant and low turbulence airflow), and the initial conditions of the drops at their releasing position were different from those expected in the natural environment (null horizontal and vertical velocity instead of the free-stream and terminal velocity, respectively), the performed validation of the coupled CFD and LPT approach on a physical fullscale model supports its application in realistic simulation scenarios.

523 Typically, applications include the numerical calculation of the collection efficiency of 524 precipitation gauges having different outer geometries (based on suitable assumptions about the 525 hydrometeors characteristics, drag coefficient and drop size distribution) to derive adjustment 526 curves for the wind-induced bias of precipitation measurements.

Previous works on this subject mostly concentrated on field studies based on the comparison of measurements obtained from shielded and unshielded gauges, and adjustment curves (also termed transfer functions) were derived from field data alone, assuming some gauge/windshield configuration as a reference. Limitations of the empirical approach include the high variability of the hydrometeor microphysical characteristics in precipitation events (Thériault et al., 2012), as well as the possible wind-induced biases associated with the reference configuration (Thériault et al., 2015). The resulting large spread of field data around the interpolating curves suggests that important driving factors are understated, such as precipitation intensity and/or the particle size distribution, as recently shown by Colli et al. (2020).

The present work aims at overcoming these existing gaps, to shed additional light on the 536 537 wind exposure problem and to achieve results that can be used operationally to adjust precipitation measurements when these are obtained in windy conditions. Indeed, the WT 538 validation of the numerical approach based on CFD and LPT simulations support its use as the 539 theoretical basis for the interpretation of the wind-induced bias of field measurements obtained 540 from shielded and unshielded precipitation gauges. Field measurements from suitable test sites, 541 equipped with reference gauges in the appropriate configuration, remains essential to provide the 542 real-world test bench and calibration basis needed to confirm the parameterization adopted (e.g. 543 for the drag coefficient) in the theoretical approach. 544

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