

An Analytical Model for CO₂ Forcing, Part II: State-Dependence and Spatial Variations

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Abstract

Clear-sky CO₂ forcing is known to vary significantly over the globe, but the state dependence which controls this is not well understood. Here we construct a quantitatively accurate analytical model for spatially-varying CO₂ forcing, which depends only on surface and stratospheric temperatures as well as column relative humidity. This model shows that CO₂ forcing is primarily governed by surface-stratosphere temperature contrast, with the corollary that the meridional gradient in CO₂ forcing is largely due to the meridional surface temperature gradient. The presence of H₂O modulates this forcing gradient, however, by substantially reducing the forcing in the tropics, as well as introducing forcing variations due to spatially-varying column relative humidity.

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ABSTRACT

11 Clear-sky CO₂ forcing is known to vary significantly over the globe, but the
12 state dependence which controls this is not well understood. Here we extend
13 the formalism of Seeley et al. (2020) to obtain a quantitatively accurate ana-
14 lytical model for spatially-varying CO₂ forcing, which depends only on sur-
15 face and stratospheric temperatures as well as column relative humidity. This
16 model shows that CO₂ forcing is primarily governed by surface-stratosphere
17 temperature contrast, with the corollary that the meridional gradient in CO₂
18 forcing is largely due to the meridional surface temperature gradient. The
19 presence of H₂O modulates this forcing gradient, however, by substantially
20 reducing the forcing in the tropics, as well as introducing forcing variations
21 due to spatially-varying column relative humidity.

22 1. Introduction

23 Changes in the Earth's CO₂ greenhouse effect (i.e. CO₂ radiative forcing) have been a primary
24 driver of past and present changes in Earth's climate, and are well simulated by state-of-the-art ra-
25 diation codes for a given atmospheric state (e.g. Mlynchak et al. 2016; Pincus et al. 2015; Oreopou-
26 los et al. 2012; Forster et al. 2011). While this accuracy is critical for credible climate simulation
27 and has thus been a priority for radiation research, less emphasis has been placed on an intu-
28 itive understanding of CO₂ forcing and its dependence on atmospheric state variables and hence
29 geography or climate. For instance, zonally averaged clear-sky CO₂ forcing exhibits a marked
30 meridional gradient (e.g. Huang et al. 2016), but what causes this? Answering such questions
31 seems particularly worthwhile given that CO₂ forcing is such a basic quantity in climate science.

32 While not very well understood, this dependence of CO₂ forcing on atmospheric state (and the
33 ensuing spatial heterogeneity of CO₂ forcing) has been known for some time and has been vari-
34 ously attributed to heterogeneities in surface temperature, lapse rate, water vapor, and cloudiness
35 (Zhang and Huang 2014; Byrne and Goldblatt 2014; Govindasamy and Caldeira 2000; Shine and
36 Forster 1999; Myhre and Stordal 1997; Kiehl and Briegleb 1993). Such studies have typically
37 still emphasized global mean forcing, however, and any attribution of the spatial structure has
38 been only qualitative. Recently, however, Huang et al. (2016) (hereafter H16) studied the spatial
39 heterogeneity of CO₂ forcing, and developed a highly accurate multilinear regression model for
40 CO₂ forcing which identified the temperature lapse rate as the most important single predictor for
41 clear-sky CO₂ forcing, followed by water vapor path. While these results point the way towards
42 understanding, such regression models cannot tell us whether their predictors have a fundamen-
43 tal significance or are simply correlated with the state variables that really matter. Furthermore,

44 such models offer limited mechanistic insight beyond that already required to sensibly choose
45 predictors.

46 Here we attempt to push our understanding further by developing a first-principles, analyti-
47 cal model for spatially varying clear-sky CO₂ forcing. The analytical model builds on the one
48 presented in the companion paper Seeley et al. (2020) (as well as that found in Wilson and Gea-
49 Banacloche (2012)) by accounting for H₂O overlap. Our analytical model is able to emulate the
50 global distribution of clear-sky radiative forcing produced by benchmark radiation codes, and the
51 simplicity of the model allows us to identify and understand the driving factors behind the geo-
52 graphical distribution of this forcing.

53 We begin in Section 2 by heuristically deriving the analytical model of Seeley et al. (2020),
54 specialized to the 500-850 cm⁻¹ spectral region. In Section 4 we use this model to compute
55 the global distribution of CO₂ forcing in the absence of H₂O for a snapshot of GCM output,
56 comparing also to a global benchmark radiation calculation. In this CO₂-only case, the analytical
57 model shows that CO₂ forcing arises from surface-stratosphere temperature contrast, and thus that
58 meridional gradients in CO₂ forcing are due almost entirely to the meridional surface temperature
59 gradient. In Section 5 we extend the analytical model to account for H₂O overlap. We again
60 compute global forcing distributions using both the analytical model and a benchmark code, and
61 find that H₂O overlap strongly modulates the meridional gradient in CO₂ forcing, by substantially
62 reducing the forcing in the tropics as well as introducing variations from spatially-varying column
63 relative humidity.

64 This work focuses on the instantaneous, top-of atmosphere (TOA), clear-sky forcing. Thus, the
65 effects of clouds (e.g. H16, Merlis 2015) as well as stratospheric adjustment (e.g. H16, Zhang and
66 Huang 2014; Stuber et al. 2001; Hansen et al. 1997; Houghton et al. 1994) are largely neglected.
67 Furthermore, the difference between instantaneous TOA forcing and instantaneous tropopause

68 forcing must be kept in mind, as the former underestimates and the latter overestimates the adjusted
 69 forcing, by as much as 40%. (e.g. H16, Zhang and Huang 2014). Consequently, the global
 70 mean instantaneous TOA forcing values shown here will be significantly lower than the somewhat
 71 canonical *adjusted* value of 3.7 W/m^2 per doubling (Ramaswamy et al. 2001; Myhre et al. 1998).
 72 The effects of clouds and stratospheric adjustment on CO_2 forcing are discussed further in Section
 73 7, and an extension of this model to instantaneous tropopause forcing is given in Seeley et al.
 74 (2020).

75 2. Theory

76 In this section we heuristically derive an analytical model for (clear-sky, instantaneous, TOA)
 77 CO_2 forcing, where CO_2 is the only radiatively active species (i.e. ‘ CO_2 -only’). This model
 78 is very similar to that from Seeley et al. (2020), except we determine our parameters somewhat
 79 differently and we treat the $500\text{-}850 \text{ cm}^{-1}$ spectral region only. This spectral region is centered
 80 roughly around the absorption peak at 667 cm^{-1} , and we will refer to it heuristically as the ‘ 667
 81 cm^{-1} band’, or simply ‘the CO_2 band’.

82 We begin with a parameterization of the spectrum of CO_2 mass absorption coefficients in the
 83 667 cm^{-1} band, evaluated at a reference pressure and temperature of $p_{\text{ref}} = 100 \text{ hPa}$ and $T_{\text{ref}} = 250$
 84 K, as

$$\kappa_{\text{ref}}(\tilde{\nu}) = \kappa_0 \exp\left(-\frac{|\tilde{\nu} - \tilde{\nu}_0|}{l}\right) \quad (1)$$

85 where $\tilde{\nu}$ denotes wavenumber (following Petty 2006; Houghton 2002; Thomas and Stamnes 2002),
 86 $\tilde{\nu}_0 = 667.5 \text{ cm}^{-1}$, $\kappa_0 \approx 60 \text{ m}^2/\text{kg}$ is a representative mass absorption coefficient at $\tilde{\nu}_0$, and the
 87 ‘spectroscopic decay’ parameter $l = 10.4 \text{ cm}^{-1}$ sets the rate at which κ_{ref} declines (exponentially)
 88 away from band center. The parameters l and κ_0 may be obtained by fitting (1) to a reference
 89 absorption spectrum produced by a line-by-line (LBL) radiation code, but the parameters turn out

90 to depend somewhat on details of the fit (e.g. Seeley et al. 2020; Jeevanjee and Fueglistaler 2020b;
 91 Wilson and Gea-Banacloche 2012). Instead, we opt to determine these parameters via optimization
 92 as described in Section 3.

93 We now write down the optical depth $\tau_{\tilde{\nu}}$ at a given wavenumber $\tilde{\nu}$, with pressure broadening but
 94 without temperature scaling (equivalent to setting $n = 1$ in Eqn. (3) of Seeley et al. 2020):

$$\tau_{\tilde{\nu}} = D\kappa_{\text{ref}}(\tilde{\nu}) \int_0^p \frac{q p'}{g p_{\text{ref}}} dp' = \frac{D\kappa_{\text{ref}}(\tilde{\nu}) q}{2g} \frac{p^2}{p_{\text{ref}}} . \quad (2)$$

95 Here q is the CO_2 specific concentration and $D = 1.5$ is a 2-stream diffusion coefficient (e.g.
 96 Clough et al. 1992). Rather than use this to solve the radiative transfer equations, however, we
 97 instead employ the ‘emission level’ approximation wherein we approximate the emission to space
 98 from CO_2 at a given wavenumber as occurring entirely at a certain emission level τ_{em} . (The
 99 emission level approximation is discussed further in Appendix B.) Setting $\tau_{\tilde{\nu}} = \tau_{\text{em}}$ in (2) and
 100 combining with Eqn. (1) then yields the ‘emission pressure’ $p_{\text{em}}(\tilde{\nu}, q)$:

$$p_{\text{em}}(\tilde{\nu}, q) = \underbrace{\sqrt{\frac{2\tau_{\text{em}} g p_{\text{ref}}}{D q \kappa_0}}}_{p_0(q)} \exp\left(-\frac{|\tilde{\nu} - \tilde{\nu}_0|}{2l}\right) . \quad (3)$$

101 The pressure $p_0(q) \equiv p_{\text{em}}(\tilde{\nu}_0, q)$ is an effective emission pressure at the center of the CO_2 band.
 102 We show in Appendix B that a suitable CO_2 emission level for our purposes is $\tau_{\text{em}}^{\text{CO}_2} = 0.5$. With
 103 this input, and for $q = 280$ ppmv, we find $p_0 = 16$ hPa, well into the stratosphere. [In reality, of
 104 course, the absorption coefficients and hence emission pressures near the center of the CO_2 band
 105 are highly wavenumber dependent (e.g. Coakley Jr. and Yang 2014) and can reach pressures lower
 106 than 16 hPa. Thus the notions of a ‘representative’ peak absorption coefficient κ_0 or an ‘effective’
 107 peak emission pressure p_0 are most definitely idealizations, but ones which will prove useful.]

108 Equation (3) can also be inverted for the wavenumbers $\tilde{\nu}_{\text{em}}$ emitting at a given p and q :

$$\tilde{\nu}_{\text{em}}^{\pm}(p, q) = \tilde{\nu}_0 \pm l \ln\left(\frac{D q \kappa_0 p^2}{2\tau_{\text{em}} g p_{\text{ref}}}\right) . \quad (4)$$

109 Note the logarithmic dependence of $\tilde{v}_{\text{em}}$ on q in this equation.

110 Figure 1a plots $p_{\text{em}}(\tilde{v})$ from Eqn. (3) for an initial CO₂ concentration $q_i = .000280 \times 44/29 =$
 111 $.000425 \text{ kg/kg}$, and for a final CO₂ concentration of $q_f = 4q_i$. Using a logarithmic axis for $p_{\text{em}}(\tilde{v})$
 112 yields triangles in the $\tilde{v} - p$ plane, with the triangle in the q_f case being taller and wider than that
 113 from q_i . Figure 1b also shows $p_{\text{em}}(\tilde{v})$ but as calculated with a benchmark line-by-line code (see
 114 calculation details in Section 3). To first order, the triangle picture is a reasonable approximation
 115 to the benchmark result.

116 The simplicity of the idealized $p_{\text{em}}(\tilde{v})$ triangles in Fig. 1a allows for a heuristic derivation of
 117 the CO₂ forcing $\mathcal{F}$ (defined as minus the difference in outgoing longwave between the q_f and q_i
 118 cases), as follows. Each orange point on the q_i curve has a corresponding point on the orange q_f
 119 curve at the same height, and thus both points have the same temperature and thus emission to
 120 space (neglecting variations in Planck function across these small spectral intervals). The orange
 121 portions of the two curves thus make identical contributions to the outgoing longwave, and thus
 122 can be neglected in calculating $\mathcal{F}$.¹ There are thus only two contributions to $\mathcal{F}$: one is the addition
 123 of new stratospheric emission from the q_f curve above $p_0(q_i)$ (dashed green), and the other is the
 124 loss of surface emission at wavelengths which were previously optically thin (solid red). In other
 125 words, the forcing is simply a swap of surface emission for stratospheric emission. This new
 126 stratospheric emission is of course what cools the stratosphere in response to increased CO₂, and
 127 it emanates from a characteristic stratospheric temperature

$$T_{\text{strat}} \equiv T \left(\sqrt{p_0(q_i)p_0(q_f)} \right) \quad (5)$$

128 where we take a geometric mean of the initial and final p_0 values to account for the roughly
 129 logarithmic dependence of T on p . For given surface and stratospheric temperatures T_s and T_{strat} ,

¹To the extent that the cooling-to-space approximation holds (Jeevanjee and Fueglistaler 2020a), this is consistent with a negligible change in tropospheric heating rate under CO₂ doubling for the CO₂-only case, as seen in e.g. Sejas et al. (2016) (their Fig. 6c).

then, their contributions to the forcing can be estimated once we know the spectral width $\Delta\tilde{\nu}$ over which these contributions are made (Fig. 1a). Using (4), we find that this effective widening of the CO₂ band from changing q_i to q_f is given by

$$\Delta\tilde{\nu} = l \ln \left(\frac{q_f}{q_i} \right) . \quad (6)$$

The logarithmic dependence of $\Delta\tilde{\nu}$ on q , which follows from (4), arises because $\tau_{\tilde{\nu}} \sim q e^{-|\tilde{\nu}-\tilde{\nu}_0|/l}$, so for fixed p and $\tau_{\tilde{\nu}} = \tau_{\text{em}}$, an arithmetic change in $\tilde{\nu}$ (which causes a uniform widening of the CO₂ band) requires a *geometric* increase in q , because the $\tilde{\nu}$ -dependence of $\tau_{\tilde{\nu}}$ is exponential. Since the forcing is proportional to $\Delta\tilde{\nu}$ (Fig. 1a), this is then the origin of the logarithmic scaling of CO₂ forcing, as derived more rigorously in Seeley et al. (2020). Also note that the overall scale of $\Delta\tilde{\nu}$ is governed by the spectroscopic decay parameter l , which governed the exponential decay of $\kappa_{\text{ref}}(\tilde{\nu})$ in Eqn. (1). Such a dual role for the spectroscopic decay parameter also occurs in the context of radiative cooling (Jeevanjee and Fueglistaler 2020b).

Returning to our derivation, if we denote the hemispherically integrated Planck function by $\pi B(\tilde{\nu}, T)$, units of W/m²/cm⁻¹, we can thus estimate $\mathcal{F}$ in this CO₂-only case as

$$\mathcal{F} = 2l \ln \left(\frac{q_f}{q_i} \right) [\pi B(\tilde{\nu}_0, T_s) - \pi B(\tilde{\nu}_0, T_{\text{strat}})] \quad (\text{CO}_2\text{-only}) . \quad (7)$$

This expression is equivalent to Eqn. (25) of Wilson and Gea-Banacloche (2012), as well as Eqn. (10) of Seeley et al. (2020) specialized to the 667 cm⁻¹ band. Note that besides the initial and final CO₂ concentrations, the only atmospheric state variables appearing in Eqn. (7) are T_s and T_{strat} . This suggests that CO₂ forcing is primarily governed by the the surface-stratosphere temperature contrast $T_s - T_{\text{strat}}$, and that the tropospheric lapse rates emphasized by H16 are only a proxy for $T_s - T_{\text{strat}}$, insofar as their vertical integral determines $T_s - T_{\text{strat}}$. This further suggests that *local* values of the tropospheric lapse rate are not relevant for CO₂ forcing, a prediction verified in

150 Seeley et al. (2020). Further physical implications of Eqn. (7) will be discussed when we study
151 spatial variations of CO₂ forcing in Section 4.

152 **3. Line-by-line calculations and parameter optimization**

153 In the remainder of this paper we will test Eqn. (7), as well as its extension to account for H₂O
154 overlap, using line-by-line radiative transfer calculations, applied to both idealized single columns
155 as well as GCM output. This section details those calculations, and uses them to optimize the
156 parameters κ_0 and l appearing in Eqns. (3) and (7), respectively.

157 *a. Line-by-line calculations*

158 Our idealized single column calculations use the Reference Forward Model (Dudhia 2017) for
159 both line-by-line spectroscopy and radiative transfer, and are very similar to the calculations per-
160 formed in Jeevanjee and Fueglistaler (2020b). We use HITRAN 2016 spectroscopic data for all
161 available spectral lines of H₂O within 0–1500 cm⁻¹ and CO₂ within 500–850 cm⁻¹, for only the
162 most common isotopologue of both gases. We consider highly idealized atmospheric profiles with
163 variable T_s and uniform tropospheric RH, a baseline CO₂ concentration of $q_i = 280$ ppmv, and a
164 constant lapse rate of $\Gamma = 7$ K/km up to to a tropopause at $T_{tp} \equiv 200$ K, with constant, negative
165 stratospheric lapse rate Γ_{strat} above. Specific humidity is uniform in the stratosphere and equal
166 to the tropopause value. For many calculations we will use a ‘BASE’ column with $T_s = 300$ K,
167 tropospheric RH = 0.75, and $\Gamma_{\text{strat}} = 0$. We run RFM at a spectral resolution of 0.1 cm⁻¹ and on a
168 vertical grid with uniform spacing of 100 m up to model top at 50 km. Calculations include H₂O
169 continuum effects unless otherwise noted, and are parameterized using RFM’s implementation of
170 the MT_CKD continuum (Mlawer et al. 2012). Far-wing absorption from CO₂ lines is suppressed
171 using RFM’s χ factor (Cousin et al. 1985). Note that we neglect the 1000 cm⁻¹ CO₂ band for

the sake of a clean comparison with Eqn. (7), but that for a quadrupling to 1120 ppmv this band can contribute an additional $\sim 1 \text{ W/m}^2$ of forcing in a global average, a roughly 10% effect (e.g. Zhao et al. 2018). See Seeley et al. (2020) and Zhong and Haigh (2013) for further analyses of the contributions from additional CO_2 bands, and their effect on the logarithmic scaling of CO_2 forcing.

The ‘global’ LBL calculations (i.e. parallelized calculations on GCM output) shown below follow those of Paynter and Ramaswamy (2012) at a resolution of 0.01 cm^{-1} , using RFM to produce optical depth profiles and then employing a four-stream solver following the method of Clough et al. (1992).

b. Parameter optimization

To set the parameters l and κ_0 and as a first, idealized test of (7), we calculate the instantaneous TOA forcing $\mathcal{F}_{4x}$ from a quadrupling of CO_2 for our idealized single columns with variable surface temperature T_s , isothermal stratosphere ($\Gamma_{\text{strat}} = 0$), and for CO_2 as the only radiatively active species (CO_2 -only). The results of this calculation, using both RFM as well as (7), are shown in Fig. 2a for various values of l . The value $l = 10.4 \text{ cm}^{-1}$ minimizes the errors in this comparison and yields an excellent fit, and will be used henceforth. Note that this value is within the $l = 10.4 - 11.5 \text{ cm}^{-1}$ range reported in Seeley et al. (2020); Jeevanjee and Fueglistaler (2020b); Wilson and Gea-Banacloche (2012), and that all values in this range yield a reasonable fit.

Next we optimize κ_0 . We do this by considering the same columns as in the previous paragraph but with $T_s = 300 \text{ K}$ and with variable Γ_{strat} . These non-isothermal stratospheres allow us to probe which κ_0 value yields the most appropriate emission pressure p_0 and hence T_{strat} [cf. Eqns. (3),(5)]. A comparison of $\mathcal{F}_{4x}$ as computed by RFM and (7) for these columns and for various values of κ_0 is shown in Fig. 2b. This panel shows that for typical values of $-4 < \Gamma_{\text{strat}} < 0 \text{ K/km}$,

the value $\kappa_0 = 60 \text{ m}^2/\text{kg}$ provides an excellent fit. This is identical to the value obtained via a fit to CO_2 *spectroscopy* (not forcing) in Seeley et al. (2020). Note, however, that large errors appear for larger magnitude Γ_{strat} , showing the limitations of using a single, idealized emission pressure p_0 to represent emission near band center. Also, note that by optimizing l first using $\mathcal{F}_{4x}$ calculations which are insensitive to κ_0 (due to $\Gamma_{\text{strat}} = 0$), and then optimizing κ_0 using this value of l , we avoid compensating errors in setting these parameter values. These and other parameter values used in this paper are tabulated in Table 1.

4. Geographic distribution of $\mathcal{F}_{4x}$ with CO_2 only

Now we apply Eqn. (7) along with (5) to more realistic atmospheric columns to obtain a geographical distribution of CO_2 forcing. We continue to consider the CO_2 -only case, postponing an analysis of the effects of H_2O overlap to Sections 5 and 6. We will also only consider forcings relative to preindustrial values of q_i . For the validity of (7) across a wider range of q_i , see Seeley et al. (2020).

We take as atmospheric data a March 22, 1981 snapshot from of a historical run of GFDL’s AM3 (Donner et al. 2011). This equinoctal snapshot has meridional temperature gradients typical of the annual mean, but also exhibits zonal variations due to synoptic-scale weather which provides a more stringent test of our simple model than annual mean fields. We calculate the forcing $\mathcal{F}_{4x}$ from a quadrupling of CO_2 for each column using both our global LBL code as well as Eqn. (7), with the results in Fig. 3a-c. Despite its simplicity, Equation (7) does an excellent job capturing the spatial pattern and overall magnitude of CO_2 forcing as calculated by the global LBL, in both the zonal mean and fully spatially resolved. The most conspicuous error is the overestimate of the zonal mean forcing near 50° N , due to a stratospheric temperature minimum near p_0 which biases our estimate of stratospheric emission.

Several other features of Fig. 3a-c deserve mention. As pointed out in the introduction and also found in previous studies (which typically include H₂O and clouds, e.g. H16, Byrne and Goldblatt 2014; Myhre and Stordal 1997), there is a strong meridional gradient in CO₂ forcing, with large values in the tropics and values close to zero or even negative near the poles. [The potentially surprising negative values² over Antarctica were emphasized by Schmithüsen et al. (2015), but subsequently put into context by Smith et al. (2018) and Flanner et al. (2018).³] There are also several small-scale regions of enhanced forcing throughout the tropics, as well as a diminished forcing over the Tibetan Plateau.

The simplicity of (7) allows us to identify the origin of these and other spatial variations in $\mathcal{F}_{4x}$. The only variables in (7) are T_s and T_{strat} , which are plotted in Fig. 3d-f. The T_s map is almost identical to the $\mathcal{F}_{4x}$ maps, showing that the spatial variations in $\mathcal{F}_{4x}$ in the CO₂-only case stem almost entirely from T_s , with T_{strat} -variations (only about ± 15 K across the globe) playing a much smaller role. Accordingly, the strong meridional gradient in zonal mean T_s matches that of $\mathcal{F}_{4x}$, while the meridional gradient in T_{strat} is weak. With such weak T_{strat} gradients, both the large-scale meridional gradient in $\mathcal{F}_{4x}$ as well as the regional features mentioned above can then be understood simply as consequences of variations in surface temperature. (In particular, the negative $\mathcal{F}_{4x}$ values over Antarctica occur because there we find $T_s < T_{\text{strat}}$.) Physically, surface temperatures are critical because they dictate the strength of the emission blocked by the widened CO₂ band (red lines in Fig. 1a).

5. Theory for $\mathcal{F}_{4x}$ including H₂O overlap

²Note that this negative CO₂ forcing is related to, but distinct from, the negative *climatological* greenhouse effect discussed in, e.g., Sejas et al. (2018).

³In particular, a negative instantaneous forcing can still lead to a positive surface temperature perturbation, because of stratospheric adjustment as well as surface-troposphere decoupling.

238 *a. Heuristics*

239 We now consider overlap between the 667 cm^{-1} CO_2 band and the H_2O rotational band and
 240 continuum. To get a feel for the impact of H_2O overlap, Fig. 4 shows the zonal mean forcing for
 241 our GCM snapshot for both the CO_2 -only and H_2O overlap cases, as computed with our global
 242 LBL code. It is immediately apparent that H_2O overlap significantly modulates the meridional
 243 gradient in CO_2 forcing from the CO_2 -only case, by significantly reducing $\mathcal{F}_{4x}$ in the tropics
 244 (H_2O overlap makes little difference in the very dry regions poleward of roughly $\pm 65^\circ$). A map of
 245 this forcing (Fig. 7a below) also shows zonal asymmetries in tropical $\mathcal{F}_{4x}$, which appear related
 246 to synoptic-scale weather.

247 To understand this, we must understand how H_2O changes the heuristic picture of CO_2 forcing in
 248 Fig. 1. Returning to our idealized single-column calculations, Fig. 5b shows $p_{\text{em}}(\tilde{\nu})$ as calculated
 249 by RFM for $q = 0, 280, 1120$ ppmv, in our BASE atmospheric column but now in the presence
 250 of H_2O . We see that the surface emission from Fig. 1 is replaced by tropospheric emission from
 251 H_2O . This should indeed reduce the forcing relative to the CO_2 -only case, as increasing CO_2 will
 252 now displace H_2O emission from the *atmosphere* rather than displacing warmer surface emission.
 253 Furthermore, this displaced H_2O emission will itself depend on relative humidity RH, and so drier
 254 areas will exhibit warmer H_2O emission and hence a stronger forcing, thus potentially explaining
 255 the meridional gradient and zonal asymmetries in tropical $\mathcal{F}_{4x}$ seen in Figs. 4 and 7a.

256 To construct an analog to Fig. 1a, we first assume that the H_2O emission on each side of the
 257 CO_2 band has an (RH-dependent) emission temperature (continuing to make the emission level
 258 approximation), and that under an increase in CO_2 it is this emission which will be swapped for
 259 stratospheric emission. This idealization is depicted in Fig. 5a. We consider the $550 - 600\text{ cm}^{-1}$
 260 spectral interval to be the low wavenumber side of the CO_2 band, and quantities averaged over

or pertaining to this interval will be signified with a ‘-’ ; similarly, we consider $750 - 800 \text{ cm}^{-1}$ as the high wavenumber side, and quantities averaged over or pertaining to this interval will be signified with a ‘+’.

To turn the heuristic picture of Fig. 5a into a formula which generalizes (7), we will estimate spectrally averaged optical depths $\tau_{\text{est}}^{\pm}$, which we can combine with an emission level $\tau_{\text{H}_2\text{O}}$ to find $T(\tau_{\text{est}}^{\pm} = \tau_{\text{em}}^{\text{H}_2\text{O}})$. (Appendix B shows that an appropriate H_2O emission level is $\tau_{\text{em}}^{\text{H}_2\text{O}} = 0.6$, which we use henceforth.) Depending on whether or not these temperatures are greater than T_s or not, we will then have emission temperatures (Eqn. (B4))

$$T_{\text{em}}^{\pm} \equiv \min(T_s, T(\tau_{\text{est}}^{\pm} = \tau_{\text{em}}^{\text{H}_2\text{O}})) . \quad (8)$$

There is an implicit but strong assumption here that in spectrally averaging Eqn. (B4), we may commute the ‘min’ function with the spectral averaging. The limitations of this assumption will become evident below. Regardless, with (8) in hand we may then construct a mean H_2O emission temperature

$$\bar{T}_{\text{em}} \equiv \frac{T_{\text{em}}^{+} + T_{\text{em}}^{-}}{2} \quad (9)$$

which can be substituted into (7) for T_s , in line with the heuristic picture in Fig. 5a.

b. Theory

Now we proceed with the quantitative details. Since optical depth is a vertical integral of absorber mass times absorption coefficient, a prerequisite for calculating $\tau_{\text{est}}^{\pm}$ is to obtain estimated, spectrally-averaged H_2O absorption coefficients $\kappa_{\text{est}}^{\pm}$. A complication, however, is that κ_{est}^{-} is dominated by line absorption, whereas κ_{est}^{+} is dominated by continuum absorption (Shine et al. 2012, and Appendix A). Accordingly, we approximate κ_{est}^{-} as scaling with foreign pressure broadening

only (Pierrehumbert 2010), whilst κ_{est}^+ scales with self-broadening only:

$$\kappa_{\text{est}}^- = \kappa_{\text{ref}}^- \frac{p}{p_{\text{ref}}} \quad (10a)$$

$$\kappa_{\text{est}}^+ = \kappa_{\text{ref}}^+ \frac{\text{RH}}{\text{RH}_{\text{ref}}} e^{(\alpha_0 - \sigma)(T - T_{\text{ref}}^+)} \quad (10b)$$

281

282 The reference absorption coefficients $\kappa_{\text{ref}}^\pm$ are evaluated at distinct reference pressures and tem-
 283 peratures $(p_{\text{ref}}^\pm, T_{\text{ref}}^\pm)$, and κ_{est}^+ also requires a reference relative humidity RH_{ref} . The constant
 284 $\alpha_0 \equiv \frac{L}{R_v T_{\text{ref}}^+{}^2}$ results from linearization of Clausius-Clapeyron, and $\sigma = .02 \text{ K}^{-1}$ is an explicit tem-
 285 perature scaling coefficient. Equation (10b) and the parameter values therein are derived in detail
 286 and evaluated in Appendix A. Parameter values are recorded in Table 1.

287 The approximations (10) then allow for an analytical evaluation of $\tau_{\text{est}}^\pm$, as follows. We integrate
 288 using temperature as our dummy integration variable, and set the lower bound of the integral to
 289 the cold-point tropopause temperature T_{tp} whose H_2O concentrations are assumed negligible (here
 290 and below we take the cold-point to demarcate the top of the troposphere). For τ_{est}^- , which we
 291 model as being due to line absorption, such a calculation was already performed in Jeevanjee and
 292 Fueglistaler (2020b), so we simply quote their Eqn. (12):

$$\tau_{\text{est}}^- = D \kappa_{\text{ref}}^- \frac{p}{p_{\text{ref}}} \text{WVP}_0 \exp\left(-\frac{L}{R_v T}\right) \quad (11a)$$

293 where $\text{WVP}_0 = \frac{(T_s + T_{\text{tp}}) \text{RH} p_v^\infty}{2\Gamma L}$ depends on RH and has units of water vapor path, $p_v^\infty = 2.5 \times 10^{11} \text{ Pa}$
 294 is a reference value for the saturation vapor pressure p_v^* where $p_v^*(T) = p_v^\infty \exp(-L/R_v T)$, and all
 295 other symbols have their usual meaning.

For τ_{est}^+ , the self-broadening scaling (10b) makes for a different calculation. Denoting vapor density by ρ_v (kg/m³) and noting that $\rho_v(T) \approx \rho_v(T_{\text{ref}}^+) \exp(\alpha_0(T - T_{\text{ref}}^+))$, we have

$$\begin{aligned}\tau_{\text{est}}^+ &= D \int_{T_{\text{tp}}}^T \kappa_{\text{est}}^+ \rho_v \frac{dT'}{\Gamma} \\ &\approx D \frac{\text{RH}^2}{\text{RH}_{\text{ref}}} \rho_v^*(T_{\text{ref}}^+) \int_{T_{\text{tp}}}^T \kappa_{\text{ref}}^+ e^{\alpha(T' - T_{\text{ref}}^+)} \frac{dT'}{\Gamma} \quad \text{where } \alpha \equiv 2\alpha_0 - \sigma \\ &= D \frac{\text{RH}^2 \rho_v^*(T_{\text{ref}}^+) \kappa_{\text{ref}}^+}{\text{RH}_{\text{ref}} \Gamma \alpha} e^{\alpha(T - T_{\text{ref}}^+)}.\end{aligned}\tag{11b}$$

Inverting Eqns. (11) at $\tau_{\text{est}}^\pm = \tau_{\text{em}}^{\text{H}_2\text{O}}$ then yields

$$T(\tau_{\text{est}}^- = \tau_{\text{em}}^{\text{H}_2\text{O}}) = \frac{T^*}{W \left[\frac{T^*}{T_{\text{ref}}^-} (D \text{WVP}_0 \kappa_{\text{ref}}^- / \tau_{\text{em}}^-)^{\frac{R_d \Gamma}{g}} \right]} \quad \text{where } T^* \equiv \frac{L R_d \Gamma}{g R_v}\tag{12a}$$

$$T(\tau_{\text{est}}^+ = \tau_{\text{em}}^{\text{H}_2\text{O}}) = T_{\text{ref}}^+ + \frac{1}{\alpha} \ln \left(\frac{\tau_{\text{em}} \Gamma \alpha \text{RH}_{\text{ref}}}{D \text{RH}^2 \rho_v^*(T_{\text{ref}}^+) \kappa_{\text{ref}}^+} \right).\tag{12b}$$

Note the dependence of T_{em}^+ on RH^2 in (12b), characteristic of the continuum. Equations (12) are the expressions we seek, and will be combined below with Eqns. (8) and (9) to yield a generalization of (7) valid in the presence of H₂O.

c. Validation

We validate the expressions (12) for $T_{\text{em}}^\pm$ by comparing them to the spectral average of

$$T_{\text{em}}(\tilde{\nu}) \equiv \min(T_s, T(\tau_{\tilde{\nu}} = \tau_{\text{em}}))\tag{13}$$

as calculated from RFM output for our single-columns with $T_s = 300$ K, no CO₂, and with varying RH. The ground truth $\int T_{\text{em}}(\tilde{\nu}) d\tilde{\nu}$ for $T_{\text{em}}^\pm$ is compared to our estimates (12) in Figure 6a,b, which shows that Eqns. (12) do an excellent job of capturing the variation of T_{em}^- with RH, and do a good job with T_{em}^+ down to RH values of 0.025, at which a significant fraction of wavenumbers in the

309 ‘+’ spectral region become optically thin and thus have $T_{\text{em}}(\tilde{\nu}) = T_s$. In this case the min function
 310 in (13) does not commute with the spectral averaging, violating the assumption behind (8). This
 311 makes our expressions for $T_{\text{em}}^{\pm}$ an overestimate whenever the relevant spectral region contains a
 312 mix of optically thick and thin wavelengths. This circumstance also occurs at the lower T_s typical
 313 of the extratropics, as we will see below.

314 [As an aside, note that the $T(\tau_{\text{est}}^{\pm} = \tau_{\text{em}})$ in Eqns. (12) are actually *independent* of T_s , i.e. they
 315 are ‘ T_s -invariant’. Once they are smaller than T_s , then, the emission to space from their respective
 316 spectral regions should also be independent of T_s . This unique property of H₂O emission temper-
 317 atures was noted as far back as Simpson (1928), and has far-reaching implications for radiative
 318 cooling and precipitation, outgoing longwave radiation, and the water vapor feedback (Jeevanjee
 319 and Romps 2018; Jeevanjee 2018; Koll and Cronin 2018; Ingram 2010). There are also implica-
 320 tions for CO₂ forcing, which we return to below.]

321 With some confidence in our expressions (12) for $T_{\text{em}}^{\pm}$, we now substitute $\bar{T}_{\text{em}}$ from Eqn. (9) into
 322 Eqn. (7) to obtain an expression for CO₂ forcing in the presence of H₂O overlap:

$$\mathcal{F} = 2l \ln \left(\frac{q_f}{q_i} \right) \left[\pi B(\tilde{\nu}_0, \bar{T}_{\text{em}}) - \pi B(\tilde{\nu}_0, T_{\text{strat}}) \right] \quad (\text{w/H}_2\text{O overlap}). \quad (14)$$

323 Note that as $\text{RH} \rightarrow 0$, $\bar{T}_{\text{em}} \rightarrow T_s$ so this equation indeed generalizes (7).

324 As a preliminary test of (14) we take our single-column, $T_s = 300$ K, variable RH calculations
 325 (with $q_i = 280$ ppmv) and compare $\mathcal{F}_{4x}$ as calculated from RFM with $\mathcal{F}_{4x}$ calculated from (14)
 326 and (12). The result is shown in Fig. 6c and shows quite good agreement between the two, though
 327 the errors in T_{em}^+ at low RH discussed above do lead to small (~ 0.5 W/m²) errors in $\mathcal{F}_{4x}$.

6. Geographic distribution of $\mathcal{F}_{4x}$ with H₂O overlap

We now estimate $\mathcal{F}_{4x}$ with H₂O overlap for our GCM snapshot using Eqns. (8), (9), (12), and (14), where Γ in Eqns. (12) is diagnosed for each column as a mass-weighted average between the cold-point tropopause and either the surface or the next highest temperature inversion, and RH is diagnosed for each GCM column as the precipitable water in the troposphere divided by its saturation value. The results of this computation are shown in Figure 7b,c, and show that Eqns. (7) and (12) capture the spatial distribution and overall magnitude of $\mathcal{F}_{4x}$ with H₂O overlap almost as well as they do in the CO₂-only case, but with slightly larger errors in the extratropics (which we discuss further below). This supports the heuristic picture of Fig. 5a, i.e. that the effect of H₂O on CO₂ forcing can be thought of as simply a change in the intensity of emission (i.e. the T_{em}) blocked by CO₂.

We now return to the T_s -invariance of $T(\tau_{est}^{\pm} = \tau_{em})$ in Eqns. (12). One consequence of this T_s -invariance, in combination with Eqn. (14), is that while CO₂-only forcing (at fixed q_i) grows with increasing T_s (Fig. 2a), forcing with H₂O overlap should asymptote to a constant value of (14) evaluated on the average of the temperatures in Eqns. (12) (assuming fixed RH and T_{strat}). We check this in Figure 8, which shows $\mathcal{F}_{4x}$ calculated from both RFM as well as Eqns. (7) and (12), for a series of our idealized atmospheric columns with variable T_s and fixed $q_i = 280$ ppmv and RH = 0, 0.75. These plots confirm that the presence of H₂O sets an upper bound on $\mathcal{F}_{4x}$ with respect to T_s which is well captured by our simple model.⁴ Physically, as T_s increases so does the water vapor path and hence the H₂O optical thickness at all wavenumbers. There is thus a transition in the origin of the emission blocked by an increase in CO₂, from surface emission to emission from H₂O, the latter of which is T_s -invariant. In reality this occurs at different water vapor paths for different wavenumbers, and thus in the spectral integral this transition is smooth

⁴A similar result can also be found in Fig. 14a of Paynter and Ramaswamy (2012).

and begins even at $T_s = 250$ K (Fig. 8a). In our simple model, however, this transition can only occur separately for the ‘-’ and ‘+’ regions (Eqns. (12)) so this transition is more discrete (Fig. 8b). Close comparison of panels a and b of Fig. 8 show that these errors in $\mathcal{F}_{4x}$ with $RH = 0.75$ maximize around $T_s = 260 - 280$ K, temperatures typical of the extratropics where the errors are also most pronounced in Fig. 7. Similar to the RH errors discussed in the previous section, these errors are due to a mix of H_2O and surface emission on either side of the CO_2 band, leading to a breakdown of the assumption behind (8) and an overestimate of the forcing.

The upper bound seen in Fig. 8 is realized beginning at roughly $T_s \approx 300$ K, a typical T_s of the present day tropics. This suggests that the forcing curves in the tropics in Fig. 7c may be thought of as having attained a global maximum (for $RH \approx 0.75$), with further local maxima in the subtropics arising only from the low RH values there. Indeed, recalculating $\mathcal{F}_{4x}$ for our GCM snapshot using (14) and (12) but fixing $RH=0.7$ yields the dashed red curve, which varies very little across the tropics. Tests also show that the zonal asymmetries in Fig. 7 are also due to zonal asymmetries in RH, due to the intrusion of deep tropical moisture filaments into the subtropics (e.g Pierrehumbert and Roca 1998; Pierrehumbert 1998).

It is important to note that this upper bound on CO_2 forcing does *not* imply an upper bound on the total greenhouse effect from CO_2 , nor on any resultant warming. Increasing CO_2 will always yield a positive forcing and hence some warming. We are making the much narrower statement that at fixed CO_2 baseline concentration q_i , the *increase* in total CO_2 greenhouse effect from increasing q_i to q_f (i.e the forcing) has a T_s -dependence which asymptotes to an upper bound. But even in the tropics where this upper bound is likely realized, every quadrupling of CO_2 will still instantaneously increase the heat trapped in the Earth system by roughly 7 W/m^2 , and that is just in the 667 cm^{-1} band. At higher q_i other spectral regions start to contribute to the forcing, making

374 this particular upper bound irrelevant and yielding a super-logarithmic scaling of CO₂ forcing with
375 CO₂ concentrations (Seeley et al. 2020; Zhong and Haigh 2013).

376 7. Summary and Discussion

377 We summarize our main results as follows:

- 378 • Clear-sky CO₂ forcing in the absence of H₂O depends solely on surface-stratosphere temper-
379 ature contrast [Eqn. (7)] and has a strong meridional gradient, which can be attributed largely
380 to the meridional gradient in surface temperature (Fig. 3).
- 381 • The meridional forcing gradient is significantly modulated by the presence of H₂O (Fig. 4),
382 where H₂O replaces surface emission at the edges of the CO₂ band with colder atmospheric
383 emission (Fig. 5).
- 384 • The T_s -invariance of H₂O emission temperatures $T_{em}^{\pm}$ implies an upper bound (with respect to
385 T_s variations) on the CO₂ forcing per doubling (Fig. 8). This upper bound is likely realized
386 in the present-day tropics (Fig. 7), but is only relevant for q_i close to preindustrial values, and
387 does not imply a saturation of the total CO₂ greenhouse effect.

388 The present work could be extended in several ways. One important extension would be to-
389 wards the calculation of the adjusted rather than instantaneous forcing, as the adjusted forcing
390 has long been recognized to be more directly related to surface warming (e.g. Hansen et al. 1997;
391 Houghton et al. 1994; Rind and Lacis 1993). Our rationale for nonetheless focusing on the sim-
392 pler, instantaneous TOA forcing here is two-fold. First, the stratospheric adjustment to the TOA
393 forcing is a 30–40% correction (H16 and Zhang and Huang 2014)), which while critical for ac-
394 curate calculation is not necessary for the basic understanding pursued here. Second, we saw
395 that spatial variations in clear-sky TOA forcing are driven largely by temperature and humidity

396 variations in the troposphere-surface system, rather than spatial variations in stratospheric temper-
397 ature; thus stratospheric adjustment is unlikely to contribute significantly to the heterogeneity or
398 state-dependence of CO₂ forcing.

399 Nevertheless, one could try to extend the formalism presented here to estimate the stratospheric
400 adjustment to the forcing. One route to doing this would be to first evaluate the instantaneous forc-
401 ing at the tropopause (rather than TOA), using the formalism presented in Appendix B of Seeley
402 et al. (2020). One could then estimate the adjusted forcing as the average of the instantaneous
403 tropopause and TOA forcings, as argued in H16.

404 Another extension of this work would be to generalize Eqn. (7) to cloudy columns, and hence
405 to compute all-sky forcing. This might be accomplished by simply replacing T_s with a diagnosed
406 cloud-top temperature, just as we replaced T_s by T_{em} in the presence of H₂O. Clouds, like H₂O,
407 should simply change the upwelling radiation which is blocked by additional CO₂. This is already
408 well-known in the literature as the ‘cloud-masking’ of CO₂ forcing (e.g. H16), but might be
409 succinctly and quantitatively described by the substitution of cloud-top temperature for T_s in (7).

410 Although this work focuses on the spatial variations of CO₂ forcing, the physics of these varia-
411 tions is simply the atmospheric state-dependence of CO₂ forcing, which also has implications for
412 CO₂ forcing as a function of base climate. For instance, a very cold, Snowball-Earth climate (e.g.
413 Hoffman et al. 2017) will have negligible H₂O and a much smaller surface-stratosphere temper-
414 ature contrast, which would lead to much reduced CO₂ forcings relative to the present day. This
415 fact and its implications for exiting the Snowball-Earth state were noted by Pierrehumbert (2004),
416 but Eqn. (7) makes this precise and allows for quantitative estimates of this effect. Note that this
417 sensitivity of Eqn. (7) to base climate means that in general one must distinguish between climate
418 sensitivity (response of T_s to given change in TOA flux) and ‘carbon sensitivity’ (response of T_s
419 to a given CO₂ change, i.e. a doubling). This distinction is also necessitated by the dependence

of CO₂ forcing on base CO₂ concentration, as emphasized in Seeley et al. (2020) and references therein.

The state dependence of CO₂ forcing may also be relevant to the spread in CO₂ forcing amongst GCMs (e.g. Soden et al. 2018; Chung and Soden 2015b,a; Zhang and Huang 2014). This spread is often attributed to parameterization error in GCM broadband radiation schemes, but may also have a contribution from spread in GCM base states. Equation (14) is computationally inexpensive to evaluate and thus might be applied to GCM output to estimate this contribution. Indeed, one can simply differentiate (7) with respect to T_s and evaluate at $T_s = 288$ K, obtaining $2l(\ln 2)\pi\frac{\partial B}{\partial T}(\tilde{v}_0, 288 \text{ K}) = 0.070 \text{ W/m}^2/\text{K}$ for CO₂ doubling. Thus, biases of 2-3 K in T_s should bias $\mathcal{F}_{2x}$ by roughly 0.1-0.2 W/m². One can also consider T_{strat} biases, which by a similar differentiation of (7) but with respect to T_{strat} and evaluated at $T_{\text{strat}} = 220$ K yields a sensitivity of 0.04 W/m²/K. Biases of 4-5 K in T_{strat} (Butchart et al. 2011) would thus also bias $\mathcal{F}_{2x}$ by 0.1-0.2 W/m².

Finally, it is worth noting that our analytical model can explain empirically-determined features of the linear regression model of H16. For example, $p_0(280 \text{ ppmv}) = 16 \text{ hPa}$ from (3) is very close to the empirically determined 10 hPa value used in H16 to evaluate stratospheric temperatures. As another example, consider H16's T_s regression coefficient of 0.066 W/m²/K for CO₂ doubling. According to our model, this coefficient should simply be the 0.070 W/m²/K calculated in the previous paragraph, a close numerical agreement.

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444 APPENDIX A

445 Estimate for H₂O self-broadened absorption coefficient

446 This appendix discusses our determination of the spectrally-averaged reference absorption coeffi-
 447 cients $\kappa_{\text{ref}}^{\pm}$ appearing in (10), and also derives the expression (10b) for the self-broadened absorp-
 448 tion coefficient profile κ_{est}^+ .

449 Self-broadened continuum H₂O absorption coefficients experience both an explicit temperature
 450 scaling as well as pressure broadening, the latter of which scales linearly with *vapor* pressure
 451 p_v rather than the dry air pressure p (Pierrehumbert 2010). These scalings are thus relative to a
 452 reference temperature and reference vapor pressure, the latter of which can be written in terms of
 453 the saturation vapor pressure p_v^* and reference RH as $p_{v,\text{ref}} = \text{RH}_{\text{ref}} p_v^*(T_{\text{ref}})$. The vapor pressure
 454 scaling can then be written

$$\frac{p_v}{p_{v,\text{ref}}} = \frac{\text{RH} p_v^*(T)}{\text{RH}_{\text{ref}} p_v^*(T_{\text{ref}})} \approx \frac{\text{RH}}{\text{RH}_{\text{ref}}} e^{\alpha_0(T-T_{\text{ref}})} \quad \text{where } \alpha_0 \equiv \frac{L}{R_v T_{\text{ref}}^2}.$$

455 As for the explicit temperature scaling, this takes the form $e^{\sigma(T_{\text{ref}}-T)}$ (Mlawer et al. 2012).

456 Since the ‘+’ wavenumber region is dominated by continuum absorption (as we will see), we
 457 will adopt the above vapor pressure scaling for κ_{est}^+ , as well as the explicit temperature scaling
 458 coefficient $\sigma = .021 \text{ K}^{-1}$ relevant for this wavenumber region (Mlawer et al. 2012). We specify
 459 reference values $\text{RH}_{\text{ref}} = 0.75$ and $T_{\text{ref}}^+ = 275 \text{ K}$ for κ_{est}^+ , and $(p_{\text{ref}}^-, T_{\text{ref}}^-) = (370 \text{ hPa}, 245 \text{ K})$ for κ_{est}^- ,
 460 which will scale with the dry air pressure [Eqn. (10a)]. In general these reference pressures and
 461 temperatures are arbitrary, and are chosen here to minimize errors from our various approximations

462 (as noted below). We can now write down κ_{est}^+ as

$$\kappa_{\text{est}}^+ = \kappa_{\text{ref}}^+ \frac{\text{RH}}{\text{RH}_{\text{ref}}} e^{(\alpha_0 - \sigma)(T - T_{\text{ref}}^+)} . \quad (\text{A1})$$

463 This is Eqn. (10b) in the main text. Equation (10a) is standard and can be found in textbooks (e.g.
464 Pierrehumbert 2010), although it neglects temperature scaling of line absorption, an issue to which
465 we return below.

466 To gauge the accuracy of Eqns. (10), Fig. A1 shows profiles of spectrally-averaged total absorp-
467 tion coefficient κ_{tot} , lines-only contribution κ_{lines} , and the difference κ_{ctm} which we can ascribe to
468 the continuum, for both the '+' and '-' wavenumber regions. These profiles are calculated via RFM
469 for our BASE column, where κ_{lines} is calculating by running RFM without continuum effects, and
470 all spectral averages are performed geometrically rather than arithmetically. Figure 7 shows that
471 for our BASE column the continuum contribution κ_{ctm} dominates in the '+' region but not in the
472 '-' region, justifying our use of continuum scalings for the '+' region only. This figure also shows
473 our estimates (10), with $\kappa_{\text{ref}}^\pm$ taken to be equal to $\kappa_{\text{tot}}^\pm(p_{\text{ref}}^\pm, T_{\text{ref}}^\pm, \text{RH}_{\text{ref}})$, yielding $\kappa_{\text{ref}}^- = 0.1 \text{ m}^2/\text{kg}$
474 and $\kappa_{\text{ref}}^+ = 0.025 \text{ m}^2/\text{kg}$. Our estimates (10) thus agree with $\kappa_{\text{tot}}^\pm$ at $(p_{\text{ref}}^\pm, T_{\text{ref}}^\pm, \text{RH}_{\text{ref}})$ by construc-
475 tion, but due to the many approximations we have made do not have the same logarithmic slope
476 (i.e. scaling) as κ_{tot} . However, because H_2O optical depth is an integral of κ_{tot} weighted by p_v ,
477 Clausius-Clapeyron scaling means it is only important for κ_{est} to have the right order of magnitude
478 in the range of $(T_{\text{em}} - 20 \text{ K}, T_{\text{em}})$ or so, within which our estimates are accurate to roughly a factor
479 of two (by our choice of $T_{\text{ref}}^\pm$).

480 It is interesting to note that the logarithmic slopes of κ_{lines} and κ_{ctm} are comparable for a given
481 wavenumber range, despite the naive expectation that κ_{lines} scales with p (which varies by a fac-
482 tor of 5 over the vertical range shown in Fig. A1) and κ_{ctm} scales with p_v (which varies by a
483 factor of 700). However, κ_{lines} also exhibits a temperature scaling, which we ignore and which

accounts for much of the error in the slope of κ_{est} in Fig. A1a. At the same time, κ_{ctm} also exhibits a temperature scaling but with opposite sign which *weakens* its C-C scaling [Eqn. (A1)]. These opposing temperature scalings for κ_{lines} and κ_{ctm} modify our naive expectations, and seem to conspire to produce surprisingly similar overall logarithmic slopes. Whether or not this is a coincidence, or is perhaps related to the hypothesis that continuum absorption is simply due to far-wing line absorption (e.g. Ma et al. 2008), could be investigated further.

APPENDIX B

On the emission level approximation and the choice of τ_{em}

In sections 2 and 5 we made the ‘emission level’ approximation that emission to space can be regarded as originating from a single level, and in the atmosphere we set these levels as $\tau_{\text{em}}^{\text{CO}_2} = 0.5$ and $\tau_{\text{em}}^{\text{H}_2\text{O}} = 0.6$. This appendix discusses this approximation, and justifies these choices of τ_{em} for our applications in particular. Other values of τ_{em} may be required for other greenhouse gases and other applications.

It will be convenient to use the framework and notation of Jeevanjee and Fueglistaler (2020a), i.e. we consider a gray gas with idealized optical depth, temperature, and source function profiles

$$\tau = \tau_s \left(\frac{p}{p_s} \right)^\beta, \quad T = T_s \left(\frac{p}{p_s} \right)^{R_d \Gamma / g}, \quad B = B_s \left(\frac{T}{T_s} \right)^\alpha$$

where subscript “s” denotes the surface value of a quantity and B has units of W/m^2 . These profiles combine to yield

$$B(\tau) = B_s \left(\frac{\tau}{\tau_s} \right)^\gamma \quad \text{where} \quad (B1)$$

$$\gamma \equiv \frac{d \ln B}{d \ln \tau} = \alpha \frac{R_d \Gamma}{g} \frac{1}{\beta}. \quad (B2)$$

Now, the emission level (EL) approximation simply says that

$$\text{OLR} \approx \begin{cases} B_s & \text{if } \tau_s < \tau_{\text{em}} \\ B(\tau_{\text{em}}) & \text{if } \tau_s \geq \tau_{\text{em}} \end{cases} \quad (\text{EL approx.}) \quad (\text{B3})$$

for some ‘emission level optical depth’ τ_{em} which may depend on the parameters introduced above.

This τ_{em} may be thought of as characterizing the transition between surface and atmospheric emis-

sion, or equivalently between ‘optically thin’ and ‘optically thick’ regimes. As such, we expect

$\tau_{\text{em}} \sim O(1)$ (Jeevanjee and Fueglistaler 2020b; Petty 2006; Wallace and Hobbs 2006), as we will

indeed find below. Note that in terms of an effective emission temperature T_{em} which satisfies

$\text{OLR} \approx B(T_{\text{em}})$, the EL approximation can be rewritten as

$$T_{\text{em}} = \min(T_s, T(\tau_{\text{em}})) . \quad (\text{B4})$$

To determine τ_{em} , we first analytically compute the OLR for our idealized gray gas, using Eqn.

(B1) and assuming $\tau_s \gg 1$:

$$\begin{aligned} \text{OLR} &= \int_0^\infty B_s(\tau/\tau_s)^\gamma e^{-\tau} d\tau \\ &= \frac{B_s}{\tau_s^\gamma} \tilde{\Gamma}(1 + \gamma) \end{aligned} \quad (\text{B5})$$

where $\tilde{\Gamma}(\gamma + 1)$ denotes Euler’s Gamma function evaluated at $\gamma + 1$, and the tilde is introduced to

distinguish it from the atmospheric lapse rate. We may then combine Eqns. (B1), (B3), and (B5)

and solve for τ_{em} , obtaining

$$\tau_{\text{em}} = \left[\tilde{\Gamma}(1 + \gamma) \right]^{1/\gamma} . \quad (\text{B6})$$

A plot of this curve is shown in Fig. 7. To determine τ_{em} , then, we simply need appropriate

values for γ for CO_2 and H_2O emission. For CO_2 , τ_{em} only enters our theory quantitatively in

determining $p_0(q)$ (Eqn. 3), which lies in the stratosphere where $\Gamma \approx -2$ K/km. Using this value

for Γ and also setting $\beta = 2$ [Eqn. (2)] and $\alpha = 4$ (Jeevanjee and Fueglistaler 2020a), Eqn. (B2)

then yields $\gamma^{\text{CO}_2} = -0.1$. Plugging this into (B6) yields $\tau_{\text{em}}^{\text{CO}_2} \approx 0.5$.

For H_2O , we are interested in tropospheric emission ($\Gamma \approx 7 \text{ K/km}$) in the neighborhood of the CO_2 band ($\alpha = 4$). Jeevanjee and Fueglistaler (2020b) found $\beta = 5.5$ for line absorption,⁵ thus yielding $\gamma^{\text{H}_2\text{O}} = 0.15$ and hence $\tau_{\text{em}}^{\text{H}_2\text{O}} = 0.6$.

It is interesting to note that the γ parameter of Eqn. (B2) was also found by Jeevanjee and Fueglistaler (2020a) to determine the validity of the cooling-to-space approximation, which was found to hold when $\gamma \ll 1$. It is also interesting to note that in this limit, we may Taylor-expand the $\tilde{\Gamma}$ function in (B6) and invoke the fact that the $\frac{d\tilde{\Gamma}(x)}{dx}|_{x=1} = -\gamma_{\text{Euler}}$, where γ_{Euler} is the Euler-Mascheroni constant (yet another gamma). A little calculation then shows

$$\lim_{\gamma \rightarrow 0} \tau_{\text{em}} = e^{-\gamma_{\text{Euler}}} = 0.56. \quad (\text{B7})$$

This gives a canonical value for τ_{em} appropriate for circumstances where $|\gamma| \ll 1$, and indeed this value is very close to both our $\tau_{\text{em}}^{\text{CO}_2}$ and $\tau_{\text{em}}^{\text{H}_2\text{O}}$.

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657		details. 36

Description	Symbol, value	Section described
Wavenumber at band maximum	$\tilde{\nu}_0 = 667.5 \text{ cm}^{-1}$	Section 2
Reference T and p for CO ₂ abs. coefficients	$(T_{\text{ref}}, p_{\text{ref}}) = (250 \text{ K}, 100 \text{ hPa})$	Section 2
Band-maximum reference CO ₂ abs. coefficient	$\kappa_0 = 60 \text{ m}^2/\text{kg}$	Section 3b
Spectroscopic decay parameter	$l = 10.4 \text{ cm}^{-1}$	Section 3b
Emission levels	$\tau_{\text{em}}^{\text{CO}_2} = 0.5, \quad \tau_{\text{em}}^{\text{H}_2\text{O}} = 0.6$	Appendix B
Reference T and p for H ₂ O abs. coefficients (different for ‘+’ and ‘-’ wavenumber regions)	$(T_{\text{ref}}^-, p_{\text{ref}}^-) = (245 \text{ K}, 370 \text{ hPa})$ $(T_{\text{ref}}^+, p_{\text{ref}}^+) = (275 \text{ K}, 650 \text{ hPa})$	Appendix A
Reference RH for cont. absorption in ‘+’ region	$\text{RH}_{\text{ref}} = 0.75$	Appendix A
Reference H ₂ O abs. coefficients	$\kappa_{\text{ref}}^- = 0.1 \text{ m}^2/\text{kg}$ $\kappa_{\text{ref}}^+ = 0.025 \text{ m}^2/\text{kg}$	Appendix A
Clausius-Clapeyron scaling coefficient in ‘+’ region	$\alpha_0 = \frac{L}{R_v T_{\text{ref}}^{+2}}$	Appendix A
Continuum T -scaling for κ_{est}^+	$\sigma = 0.021 \text{ K}^{-1}$	Appendix A
T -scaling for τ_{est}^+	$\alpha = 2\alpha_0 - \sigma$	Appendix A

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- Fig. 6.** (a,b) Validation of our simple expressions (12) for band-averaged H₂O emission temperatures $T_{\text{em}}^{\pm}$, as compared to band-averaged $T(\tau_{\tilde{\nu}} = \tau_{\text{em}})$ using $\tau_{\tilde{\nu}}$ as output by RFM. This comparison is made for idealized atmospheric columns with $T_s = 300$, no CO₂, and varying RH. (c) Validation of the simple model of Eqns. (12) and (7) for $\mathcal{F}_{4x}$ in the presence of H₂O, as compared to $\mathcal{F}_{4x}$ calculated by RFM. This comparison is made for idealized atmospheric columns with $T_s = 300$, $q_i = 280$ ppmv, and varying RH. The simple expressions (12) predict $T_{\text{em}}^{\pm}$ very well except at low RH in the ‘+’ region, leading to small ($\sim 0.5 \text{ W/m}^2$) errors in $\mathcal{F}_{4x}$ at these RH values. 44
- Fig. 7.** As in Fig. B3a-c, but now including H₂O overlap, where panel (b) is generated using Eqn. (14). By replacing warm surface emission with colder tropospheric emission, H₂O strongly modulates the CO₂ forcing outside the dry polar regions. Spatial variations in $\mathcal{F}_{4x}$ within the tropics due to strong RH variations can also be seen. These features are emulated by the simple model. 45

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 707 bound on $\mathcal{F}_{4x}(T_s)$ (with respect to T_s variations and at fixed baseline CO₂) which does not
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 714 of line absorption, but is acceptably close within 20 K or so of T_{ref}^- . The horizontal axis in
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 717 At $\gamma = 0$, $\tau_{\text{em}} = e^{-\gamma_{\text{Euler}}} \approx 0.56$, giving a canonical value for τ_{em} which is indeed close to our
 718 value for $\tau_{\text{em}}^{\text{CO}_2}$ and $\tau_{\text{em}}^{\text{H}_2\text{O}}$ 48

Emission levels, CO₂ only

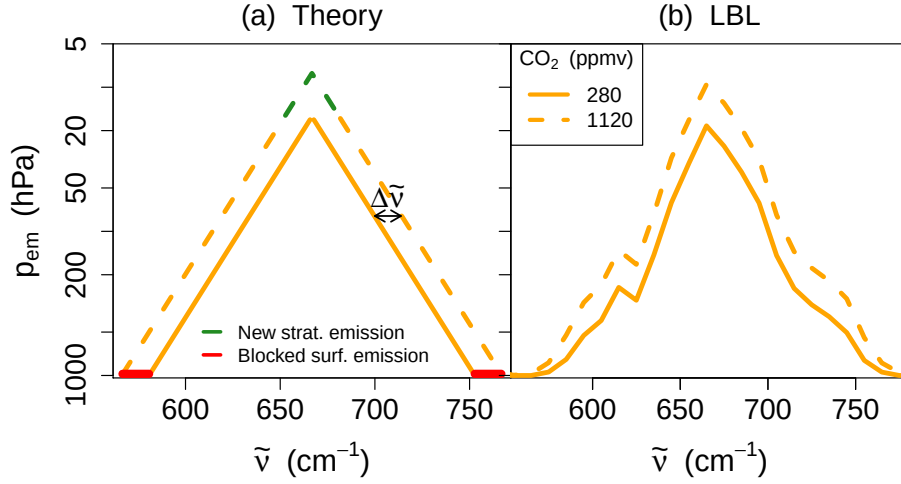


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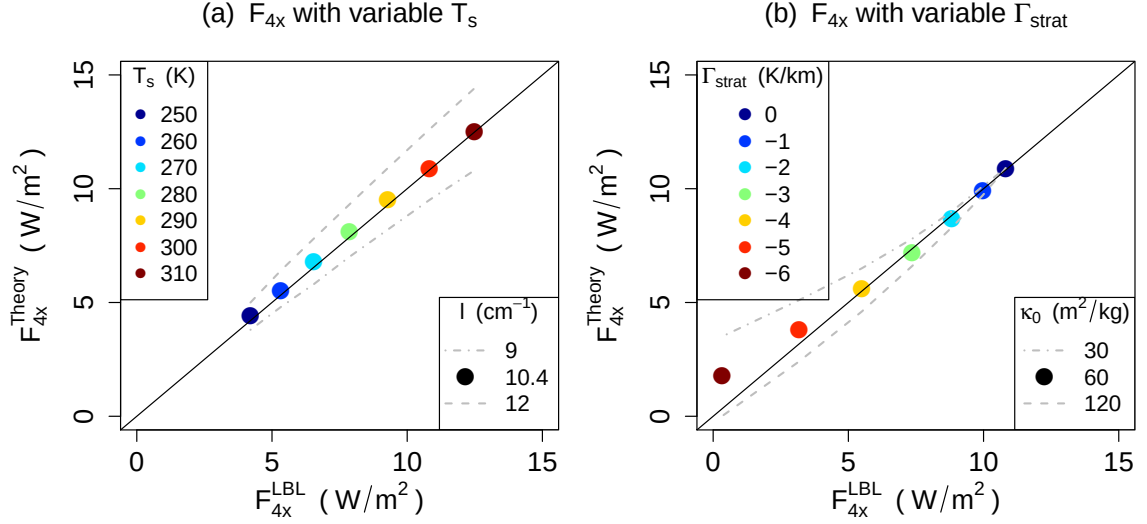


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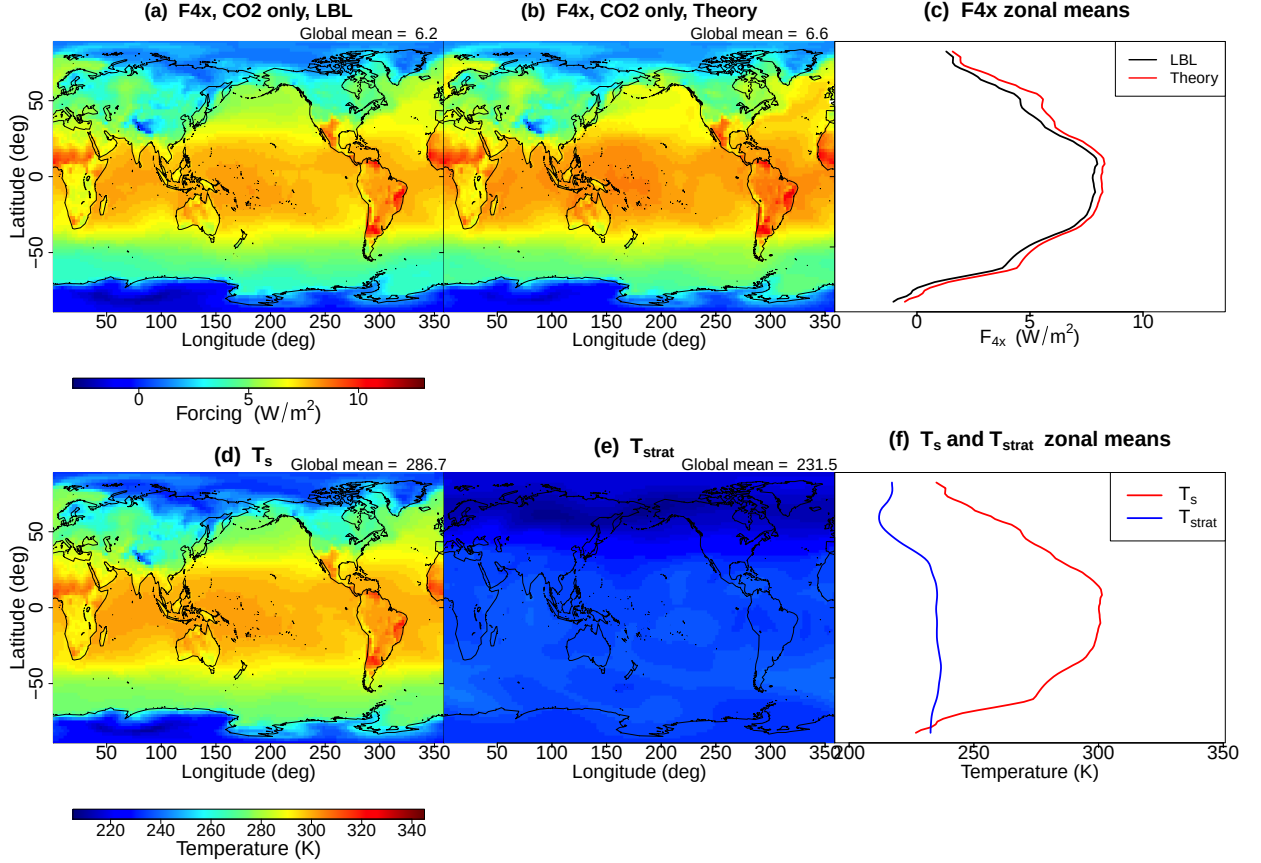


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LBL forcing, w/ and w/o H2O overlap

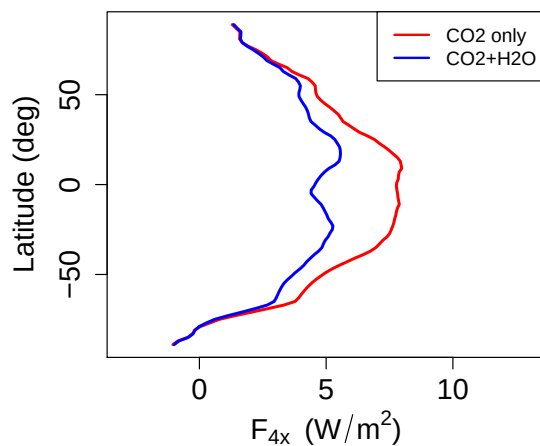


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Emission levels, CO₂ + H₂O

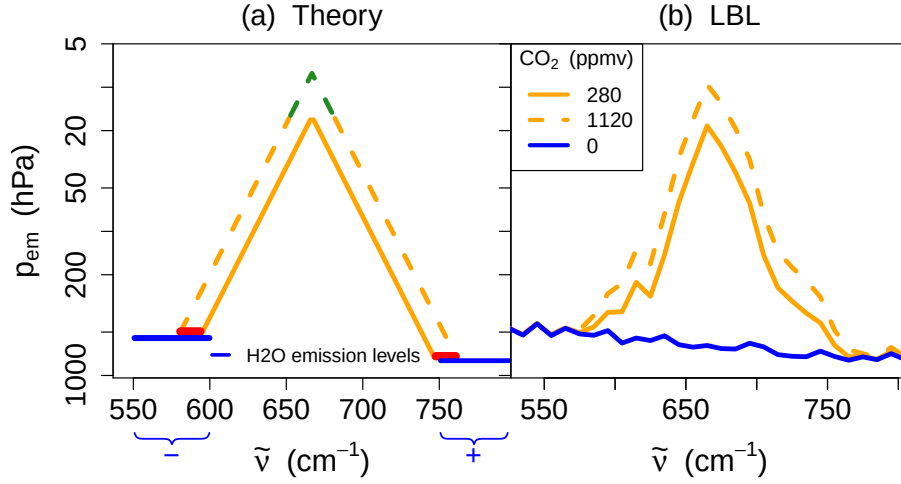


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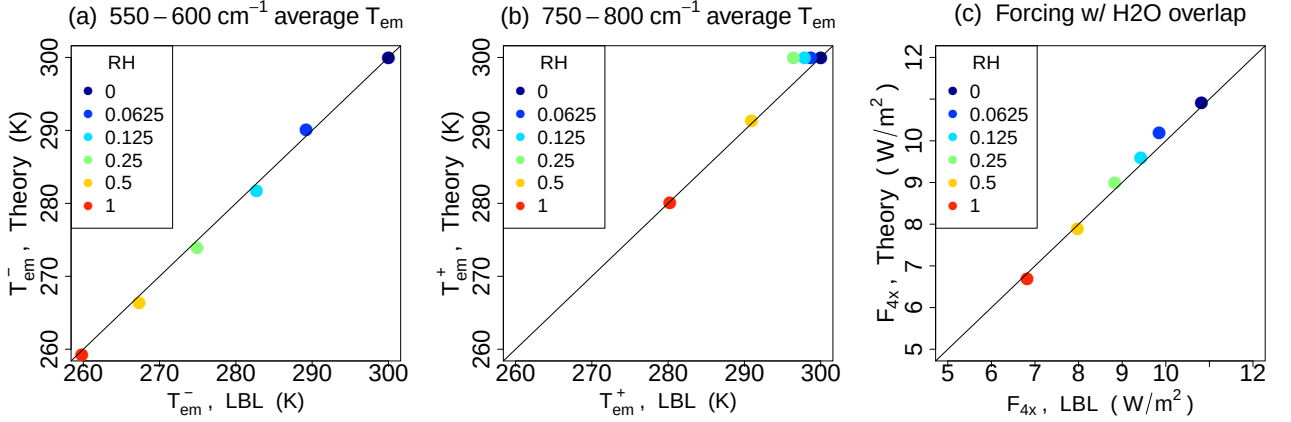


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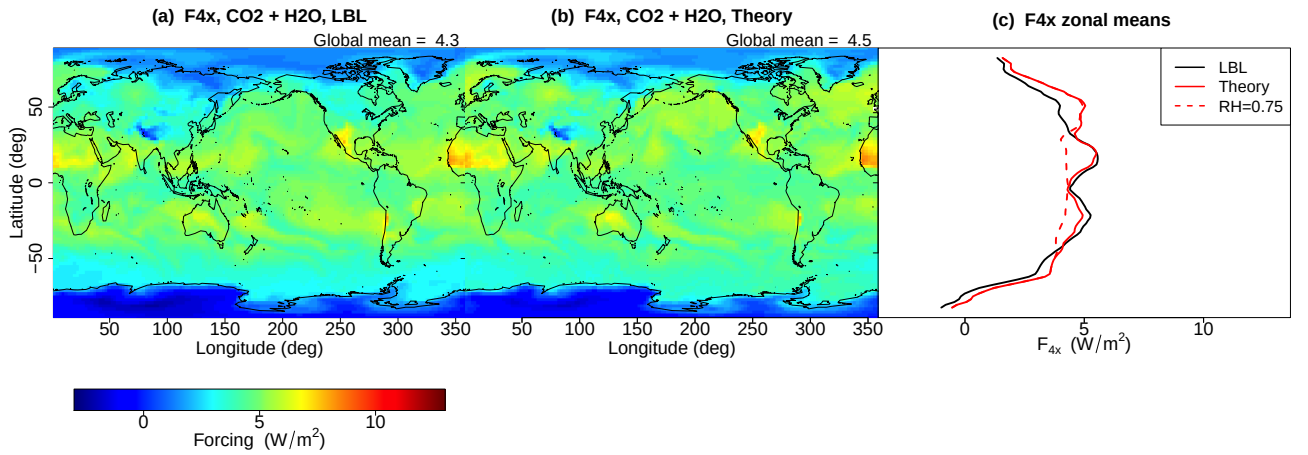
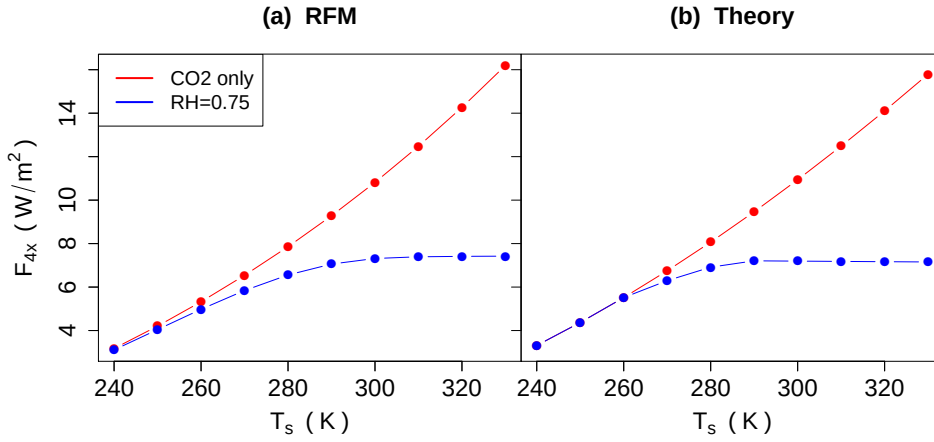


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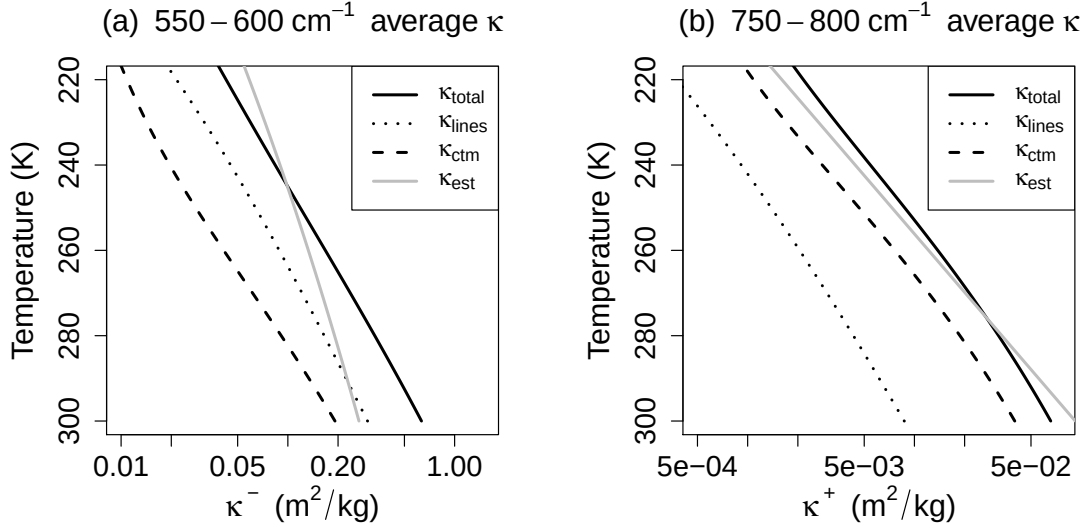


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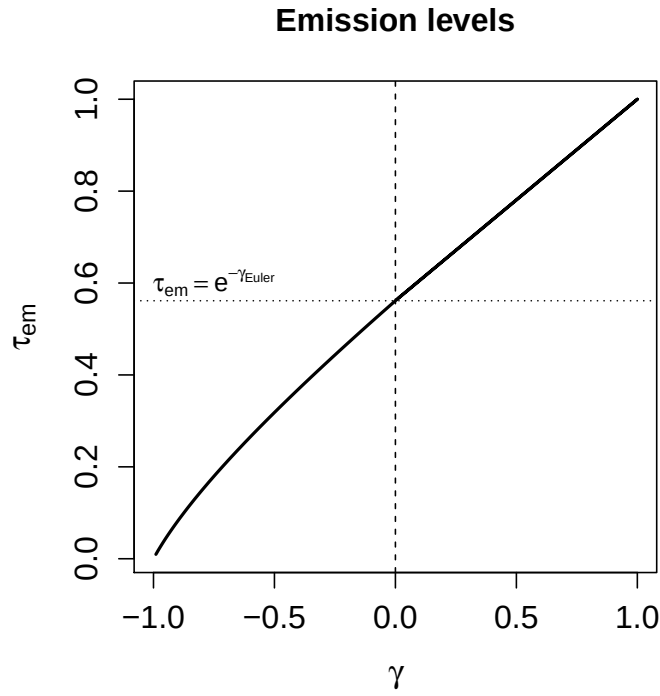


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