The internal structure of Mercury's core inferred from magnetic observations

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Abstract

Previous models of Mercury's core magnetic field based on high altitude data from first MESSENGER flybys revealed an axisymmetric. Here we use low altitude MESSENGER data covering the entire mission period to construct spherical harmonic models based on various spatial norms. Although we find a dominantly axisymmetric field, our models nevertheless include detectable deviations from axisymmetry. These non-axisymmetric features appear at high latitudes, resembling intense geomagnetic flux patches at Earth's core-mantle boundary. Based on this core field morphology, we then attempt to infer Mercury's internal structure. More specifically, assuming that Mercury's high-latitude non-axisymmetric features are concentrated by downwellings at the edge of the planet's inner core tangent cylinder, and accounting for the presence of a stably stratified layer at the top of Mercury's core, we establish a relation between the inner core size and the thickness of the stratified layer. Considering plausible ranges, we propose that Mercury's inner core size is about 500-660 km, which corresponds to a stratified layer thickness of 880-500 km, respectively.

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9	Key Points:
10	• We model Mercury's internal magnetic field from MESSENGER data with spher-
11	ical harmonics
12	• Our core field model contains non-axisymmetric features from which we make in-
13	ferences of Mercury's internal structure.
14	- We estimate Mercury's inner core radius of \sim 500-660 km and a corresponding thick-

ness of a top stratified layer of ${\sim}880\text{-}500$ km.

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16 Abstract

Previous models of Mercury's core magnetic field based on high altitude data from first 17 MESSENGER flybys revealed an axisymmetric. Here we use low altitude MESSENGER 18 data covering the entire mission period to construct spherical harmonic models based 19 on various spatial norms. Although we find a dominantly axisymmetric field, our mod-20 els nevertheless include detectable deviations from axisymmetry. These non-axisymmetric 21 features appear at high latitudes, resembling intense geomagnetic flux patches at Earth's 22 core-mantle boundary. Based on this core field morphology, we then attempt to infer Mer-23 cury's internal structure. More specifically, assuming that Mercury's high-latitude non-24 axisymmetric features are concentrated by downwellings at the edge of the planet's in-25 ner core tangent cylinder, and accounting for the presence of a stably stratified layer at 26 the top of Mercury's core, we establish a relation between the inner core size and the thick-27 ness of the stratified layer. Considering plausible ranges, we propose that Mercury's in-28 ner core size is about 500-660 km, which corresponds to a stratified layer thickness of 29 880-500 km, respectively. 30

31 1 Introduction

Based on Mariner and MESSENGER satellite missions, it was found that Mercury's 32 internal magnetic field is very weak, dipole dominated, largely axisymmetric and with 33 a magnetic equator shifted northward with respect to the geographic equator at mid-34 latitudes of the northern hemisphere (Ness et al., 1974; Ness, 1979; Anderson et al., 2011, 35 2012; Johnson et al., 2012; Oliveira et al., 2015; Thébault et al., 2018; Wardinski et al., 36 2019), which is challenging to explain in terms of core structure and dynamics. In ad-37 dition, several studies of Mercury's interior based on analyses of MESSENGER gravity 38 field measurements and libration data suggested that the top of Mercury's outer core is 39 thermally stratified (Smith et al., 2012; Dumberry & Rivoldini, 2015). Likely, this layer 40 is comprised of FeS, but its phase (liquid or solid) remains uncertain. Thermal strati-41 fication implies that the heat flux at the core surface is sub-adiabatic, which has impor-42 tant implications for the inner core solidification and the magnetic field generation. 43

Numerical dynamo simulations may provide further insight into Mercury's core structure. Possible scenarios include deep-seated dynamos below a thick stable layer (Christensen, 2006; Christensen & Wicht, 2008; Takahashi et al., 2019), thin-shell dynamos (Stanley et al., 2005) and dynamos with a thin stratified layer (Stanley & Mohammadi, 2008).

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The weakness of Mercury's core field motivated modeling a sulfur-rich liquid core with 48 different zones of Fe-precipitation, i.e. iron snow. These zones could exist at the bottom 49 of the liquid core or at the mid range between the inner and outer core boundaries (Vilim 50 et al., 2010). While numerical dynamos with a stably stratified layer at the top of the 51 shell can explain Mercury's weak magnetic field and its axisymmetry, additional ingre-52 dients are needed to explain the northward shift of the magnetic equator. Numerical dy-53 namos with imposed heterogeneous heat flux in the form of equatorial cooling at the outer 54 core boundary (Cao et al., 2014) lead to a convective instability and an offset of the mag-55 netic equator, but their magnetic fields are too energetic and non-axisymmetric. In con-56 trast, Tian et al. (2015) imposed a degree-1 axially heterogeneous heat flux on a dynamo 57 model with a stratified layer at the top of the shell and hyper-diffusivity to obtain more 58 Mercury-like magnetic fields. In both cases the validity of the results relies on the ac-59 tual pattern of thermal heterogeneity at the base of Mercury's mantle, which is largely 60 uncertain. Double diffusive convection phenomena have also been considered to explain 61 Mercury's magnetic field. These phenomena occur when the convection is driven by two 62 sources of buoyancy, i.e. temperature and composition (Manglik et al., 2010). Recently, 63 Takahashi et al. (2019) showed that a double-diffusive convecting shell surrounded by 64 a thick thermally stably stratified layer can generate Mercury-like magnetic fields. Fur-65 thermore, numerical dynamo models of Mercury's magnetic field provide estimates of the 66 size of Mercury's inner core. Cao et al. (2014) suggested an inner core radius smaller than 67 1000 km. Based on geodetic analyses Dumberry and Rivoldini (2015) gave an upper limit 68 on the inner core size of 650 km, with the outer core dynamics partly consisting of snow 69 formation. 70

These different scenarios of Mercury's dynamo lead to characteristics that should
 be testable by observations of space-borne magnetometers like MESSENGER and Bepi Columbo. A careful processing and analysis of magnetic field measurements taken in plan etary environments is crucial for the identification of such magnetic field characteristics.

⁷⁵ Mainly, two techniques have been applied to study Mercury's magnetic fields: po-⁷⁶ tential field methods such as spherical harmonics (Uno et al., 2009; Wardinski et al., 2019), ⁷⁷ spherical caps (Thébault et al., 2018) or equivalent source dipoles (Oliveira et al., 2015) ⁷⁸ that restrict the analysis to those observations obtained in a source free region, and (re-⁷⁹ duced) parametric models that infer the magnetic dipole moment from the space probe's ⁸⁰ magnetic equator crossing, i.e. where the radial field B_r is zero, far from the planet (An-

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derson et al., 2012; Johnson et al., 2012). The latter method provides models with a re-81 duced set of parameters and is popular because of its relative independence of the data 82 distribution. Data used for these reduced parametric models sampled the magnetic field 83 in the magnetospheric region, with a considerable electrical current density that requires 84 additional assumptions about the geometry and distribution of local current systems (Con-85 nerney & Ness, 1988). In contrast, Uno et al. (2009) showed by inverting synthetic data 86 from numerical dynamo simulations that a spherical harmonic analysis can recover the 87 large-scale magnetic field from hemispherical uneven data distribution, as single MES-88 SENGER flybys, when data are taken in a source-free region. The resolution of finer de-89 tails of the magnetic field needs, off course, numerous orbital tracks. 90

In this study magnetic field data are used to derive field models that may constrain 91 the internal structure of a planet. The downward continuation of a magnetic field model 92 to the core surface reveals patterns of magnetic flux. In particular, the latitude at which 93 intense flux patches are concentrated may indicate the size of the inner core. Intense flux concentrations near the intersection of the inner core tangent cylinder are prominent in 95 the geomagnetic field for at least the last 400 years (e.g. Jackson et al., 2000) and possibly over the past tens of millennia (see Panovska et al., 2019, and references therein). 97 Numerous studies explored the kinematics as well as the dynamical origin of intense high-98 latitude flux patches in geomagnetic field models and numerical dynamos (Bloxham et 99 al., 1989; Christensen et al., 1998; Amit et al., 2010, 2011; Peña et al., 2016; Olson et 100 al., 2018). The latitude at which these flux concentrations occur has been related to the 101 change of the dynamical regime at the tangent cylinder that is coaxial with the rotation 102 axis and tangential to the inner core boundary (Gubbins & Bloxham, 1987), while the 103 longitude at which these flux patches occur may be controlled by thermal core-mantle 104 interactions (Bloxham & Gubbins, 1987). 105

Here we will use inferences from the Earth's core to carefully establish the relation between the latitude of intense magnetic flux patches and the tangent cylinder intersection with the core-mantle boundary (CMB), including possible errors associated with time-dependence and variability from one patch to another. We will then account for the existence of stratification to relate the depth of the stable layer with the radius of the inner core for a given latitude of magnetic flux patches. This relation will be implemented for the case of Mercury's magnetic field.

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The aim of this study is twofold: First, we explore to what extent intermediate-113 scale spatial features of Mercury's magnetic field can be retrieved from the MESSEN-114 GER data by applying a spherical harmonic analysis; Second, we aim to infer the inter-115 nal structure of Mercury's core and the convective state of its dynamo. The paper is or-116 ganized as follows: The description of the data and their selection is given in section 2, 117 section 3 briefly describes the spherical harmonic modeling method, and results are pro-118 vided in section 4. Implications for the generation of Mercury's core field and the struc-119 ture of its core are discussed in section 5. We summarize our main findings in section 120 6. 121

122 **2** Data selection

The MESSENGER spacecraft was in orbit around Mercury from 18 March 2011 to 30 April 2015. The orbit of MESSENGER was highly eccentric, with periapsis ranging from 200 to 500 km over the north polar region, and apoapsides of > 12700 km above the southern hemisphere. This highly eccentric orbit led to an uneven data distribution, where only measurements over the northern hemisphere are assumed to be inside the magnetospheric cavity which allow adequate modeling of Mercury's internal magnetic field (Oliveira et al., 2015). All local times are covered within 88 (terrestrial) days.

Here, we selected data from a satellite altitude range of 300 to 1000 km during lo-130 cal night-time. The night-time selection criterion is often used in the derivation of ge-131 omagnetic field models that are based on satellite data (Finlay et al., 2016; Lesur et al., 132 2008; Olsen et al., 2006). This has proven to provide data sets with largely removed ex-133 ternal field contamination and to allow a precise description of Earth's core field to high 134 spherical harmonic degrees. To this aim, we apply the same selection criterion to the MES-135 SENGER data set. The altitude selection criterion guarantees that the analyzed mag-136 netic field measurements are within the magnetospheric cavity: 1000 km is smaller than 137 the averaged subsolar distance of the magnetopause location (Winslow et al., 2013; Thébault 138 et al., 2018), while the lower limit excludes data from the beginning and the end of the 139 MESSENGER mission. Oliveira et al. (2019) showed that the crustal magnetic signal 140 is small-scale, and weak in amplitude at 40 km altitude. Therefore, at 300 km altitude 141 signals due to crustal magnetization are assumed to be negligible at large length scales. 142 The combination of both criteria provides a data set which shows no crustal magnetic 143

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- signatures and the least contamination from magnetospheric and exospheric magnetic
- fields (Wardinski et al., 2019), which are strong at the planet's day-side.

¹⁴⁶ 3 Method

All of the selected data over the northern hemisphere (see section 2) sampled Mercury's magnetic field in a region with almost no magnetic sources. Therefore, a potential theory and spherical harmonic analysis provides adequately a seperation between external and internal magnetic field sources. We seek to fit MESSENGER observations of Mercury's magnetic field by a potential that is parameterized using spherical harmonics, i.e.

$$V = a \sum_{l=1}^{L_{\text{int}}} \sum_{m=0}^{l} \left\{ (g_l^m \cos(m\phi) + h_l^m \sin(m\phi)) \left(\frac{a}{r}\right)^{l+1} P_l^m (\cos\theta)) \right\} + a \sum_{l=1}^{L_{\text{ext}}} \sum_{m=0}^{l} \left\{ (q_l^m \cos(m\phi) + s_l^m \sin(m\phi)) \left(\frac{r}{a}\right)^l P_l^m (\cos\theta) \right\},$$
(1)

where a is Mercury's radius (2440 km). r is the radial distance from Mercury's center, 153 θ the colatitude and ϕ the longitude. $P_l^m(\cos\theta)$ are the Schmidt semi-normalized asso-154 ciated Legendre functions, where l is the degree and m the order. L_{int} and L_{ext} are the 155 truncation degrees of the spherical harmonic expansions for the internal and external field, 156 respectively. The Gauss coefficients $\{g_l^m,h_l^m\}$ and $\{q_l^m,s_l^m\}$ represent the internal and 157 external magnetic field, respectively. These model parameters are estimated by a least 158 squares fit to data collected during a given time interval. In the following, we outline de-159 tails of our modeling technique, which is sometimes called smoothed inversion (Holme 160 & Bloxham, 1996; Uno et al., 2009). From the selected data, we derive models with $L_{\rm int} =$ 161 10 and $L_{\text{ext}} = 1$. External magnetic fields of higher spherical harmonic degrees can-162 not be estimated with confidence by using a regularized inversion. Their signals should 163 contribute to the model residuals, i.e. un-modeled signals. These un-modeled signals could 164 be partly related to the magnetic signatures of Birkeland currents, which mainly exist 165 in the dawn and dusk sections of the data local times at latitudes higher than 70° North. 166 Their signals are generally in the horizontal field components, with magnitudes of only 167 20 nT and they do not rotate with the planet (Anderson et al., 2018). 168

¹⁶⁹ **3.1 Model priors**

For a linear least squares problem the model vector **m** containing the Gauss coefficients is found at the minimum of an objective function

$$\Theta(m) = (\mathbf{y} - \mathbf{A}\mathbf{m})^{\mathsf{T}} \mathbf{C}_{\mathbf{e}}^{-1} (\mathbf{y} - \mathbf{A}\mathbf{m}) + \lambda_{S} (\mathbf{m}^{\mathsf{T}} \mathbf{C}_{\mathbf{m}}^{-1} \mathbf{m}), \qquad (2)$$

where y is the data vector, **A** a design matrix, C_e the data error covariance matrix and C_m the prior model covariance matrix (Jackson, 1979; Gubbins, 1983) which is controlled by a Lagrange multiplier (λ_S). The misfit of the model is computed by

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{N} (y_i - \hat{y}_i)^2}{N - 1}},$$
(3)

where \hat{y}_i is the model value for given observation y_i .

Our method to find the Gauss coefficients utilizes prior constraints to reduce the ambiguity of the data inversion. The application of priors to constrain the inversion of MESSENGER data to obtain a model of Mercury's magnetic field is justified by their uneven hemispherical distribution. Primarily, the prior should emphasize the large-scales of Mercury's magnetic field. We test the performance of four different priors of the spatial complexity of the model's field morphology. These priors are usually formulated as model norms:

Norm 1:
$$\oint B^2 dS|_{r=c} = (l+1) \left(\frac{a}{c}\right)^{(2l+4)}$$

Norm 2: $\oint B_r^2 dS|_{r=c} = \frac{(l+1)^2}{2l+1} \left(\frac{a}{c}\right)^{(2l+4)}$
Norm 3: $\oint (\nabla_h B_r)^2 dS|_{r=c} = \frac{l(l+1)^3}{2l+1} \left(\frac{a}{c}\right)^{(2l+6)}$
Norm 4: $\oint F dS|_{r=c} = \frac{(l+1)(2l+1)(2l+3)}{l} \left(\frac{a}{c}\right)^{(2l+3)} \leq Q.$
(4)

In Norm 4 of (4) F is the field intensity and Q is the mean CMB heat flux. In all these 183 expressions (4), Mercury's core radius is c = 2060 km (Wardinski et al., 2019). Norm 184 1 minimizes the power of the magnetic field for higher spherical harmonic degrees, and 185 therefore it steepens the slope of its power spectrum. Norms 2 and 3 smooth the radial 186 magnetic field and its horizontal gradient, respectively (Shure et al., 1982). Norm 4 is 187 different from the other norms as it may include prior knowledge of the heat flux at Mer-188 cury's core surface, which is due to the Ohmic dissipation of the radial field B_r at the 189 core surface (Gubbins, 1975). However, there are no observations of the heat flux at Mer-190 cury currently available. Therefore, Norm 4 acts merely as a constraint to stabilize the 191

solution of the inversion like the other norms. All norms dim the amplitude of small-scale 192 field features, though at different ways, hence support the large-scale morphology of the 193 magnetic field. Among these norms, this effect is most strongly imposed by Norm 3, where 194 the attenuation scales with l^3 . 195

- The resulting model is determined by varying the strength of the prior to be in op-
- timal balance between data misfit and model smoothness. This optimal balance is usually found for the λ_S at the knee of their trade-off or L-curves.
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3.2 Iterative modeling scheme

To find the model parameters, we adopt an iterative re-weighting scheme that con-200 sists of three steps. At a first step, we determine a model that is based on data cover-201 ing the MESSENGER's entire mission interval at Mercury. Data are weighted equally, 202 to form the initial error covariance matrix, C_e , in (2). We assign an initial error of 1.6 203 nT to each datum which corresponds to the upper limit of the instrument's resolution 204 (Anderson et al., 2007). At a second step, individual differences between each data and 205 corresponding values of the initial model are computed, to provide an update of C_e and, 206 third, to derive the final model with the updated error covariance matrix. The residual 207 amplitude, and therefore $\mathbf{C}_{\mathbf{e}}$, depends directly on the Lagrange multiplier λ_S ; In order 208 to obtain a close trade-off curve for each norm, this iterative re-weighting scheme is ap-209 plied for each setting of λ_s . In total we derive a large number of models for each norm. 210 We select the model at the knee of each norm trade-off curve. 211

A closer inspection of the residuals reveals anomalous tracks that show significant 212 larger residual amplitudes than others. The cause for these large residuals remains un-213 clear, but could be related to instrument errors and/or data processing errors. However, 214 these data are automatically down-weighted and rejected from the model derivation when 215 the misfit is larger than 2σ . 216

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3.3 Robustness of the solutions

There are a few diagnostics to evaluate the robustness and confidence of the inver-218 sion results. First, we analyze the resolution matrix of the model m, to obtain a mea-219 sure of model parameter significance. The resolution matrix is given by 220

$$\mathbf{R} = (\mathbf{A}^{\mathsf{T}} \mathbf{C}_{\mathbf{e}}^{-1} \mathbf{A} + \lambda_{s} \mathbf{C}_{\mathbf{m}}^{-1})^{-1} \mathbf{A}^{\mathsf{T}} \mathbf{C}_{\mathbf{e}}^{-1} \mathbf{A}, \qquad (5)$$

	Norm1	Norm 2	Norm 3	Norm 4
$\overline{\lambda_s}$	8.0×10^{2}	8.0×10^{2}	4.0×10^{2}	8.0×10^{6}
rms misfit (nT)	26.11	26.09	26.5	26.39
Trace of R-Matrix	65	70	36	57
rms field intensity (nT)	331.19	332.75	327.05	335.24
$g_1^0 (\mathrm{nT})$	-217.5	-218.8	-213.8	-221.5
g_2^0/g_1^0	0.29	0.29	0.31	0.28
dipole tilt angle (°)	0.6	0.4	0.7	0.8

 Table 1.
 Inversion parameters, diagnostics and global characteristics of the field models.

where the diagonal elements of $\mathbf{C}_{\mathbf{m}}$ are defined in (4). Ideally, this matrix would be an 221 identity matrix. Due to inadequacy of the data, a regularization scheme must be applied 222 in the solving process to obtain a stable solution. This is reflected in the form of the res-223 olution matrix. A value of the resolution near 1 means that a model parameter is wholly 224 determined by the data, whereas a low resolution, i.e. values ~ 0.1 , means that the model 225 is mostly controlled by the prior information. The trace of the resolution matrix $Tr(\mathbf{R})$ 226 can be broadly interpreted as the degree of freedom of the model and as the number of 227 model parameters resolved by the inversion (Tarantola, 1987). 228

Characteristics and diagnostics of the field models are listed in Table 1. All models widely agree in their statistical properties, and mostly differ in their numbers of resolved parameters. The lowest number of resolved parameters is found for Norm 3, as it more strongly damps contributions of higher spherical harmonic degrees than other norms and therefore reduces the degree of freedom most strongly.See appendix B for a further discussion of the inversion covariance matrix.

235 4 Results

In this section, we present models of Mercury's time-averaged magnetic field as they are based on MESSENGER measurements covering the period 2011-2015. We discuss to what extent our results are conclusive and estimate their robustness. The models provide reasonably good fits to the data with residuals of $\sim 8\%$ of the total field strength.

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Figure 1. Diagonal elements of the resolution matrix of the preferred models for different priors vs. coefficient number (see also degree at the top axis). Gray shaded areas indicate even spherical harmonic degrees. The colored vertical lines represent the degree of freedom of the respective model.

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4.1 Resolution analysis and spectral content

For each of the four resulting models we computed the resolution matrix \mathbf{R} , and charted their diagonal elements, where labels of the models refer to the norm used to constrain the solution, e.g. Norm $1 \rightarrow$ Model 1. These plots (resolution curves) are shown in Figure 1.

The resolution curves of all models generally agree for the first four spherical har-245 monic degrees. Most notable are the high resolution of sectorial Gauss coefficients, i.e. 246 g_m^m, h_m^m , of at least the first five spherical harmonic degrees. Resolution curves of mod-247 els 1, 2 and 4 show this particular pattern also for higher spherical harmonic degrees, 248 which may indicate a higher ability of these norms to capture small-scale magnetic field 249 signatures. The higher resolution of the sectorial terms could also be explained by MES-250 SENGER's flight path along latitude. We find that Model 3 resolves magnetic field struc-251 tures only until spherical harmonic degree 5. Model 4 partly resolves spherical harmonic 252 degree 7, whereas Models 1 and 2 show a higher resolution and partly resolve degree 8. 253 Vertical lines in Figure 1 mark the maximum number of resolved model parameters cor-254

responding to the trace of the resolution matrix (see Table 1). This coincides with a res-255 olution level of ~ 0.4 . Below this level, the model is assumed to be dominated by the 256 prior. At degree 10, i.e. coefficient numbers between 100 and 120, Models 1, 2 and 4 show 257 an enhanced resolution, where they may become sensitive to spectral leakage, signals of 258 other sources and possible data errors. We assume the high resolution at these small scales 259 to be an artifact and possibly caused by the orbital geometry of MESSENGER. Holme 260 and Bloxham (1996) discussed a similar effect observed in the Voyager II data at Nep-261 tune, which was likely caused by the spacecraft trajectory. The setting of a maximum 262 degree L_{int} for the spherical harmonic expansion leads also to an aliasing of the higher 263 degree field $(l > L_{int})$ into coefficients of the model spherical harmonic expansion (spec-264 tral leakage). We assume this effect to be reduced by truncating the models to spherical 265 harmonic degree $L_{\text{int}} = 8$. 266

Maps derived from truncated models (l = 8) of the radial magnetic field at the 267 core surface are shown in Figure 2. As expected field structures in the northern hemi-268 sphere show more details than in the southern hemisphere, because of the data distri-269 bution. Generally, the field is dominated by the axial dipole, but the magnetic equator 270 is significantly shifted towards the North pole in agreement with previous studies (An-271 derson et al., 2011, 2012; Thébault et al., 2018). The different models show differing spa-272 tial complexities, in particular of the magnetic equator. The map derived with Norm 3 273 shows the most axisymmetric field morphology, whereas models derived with Norms 1 274 and 2 show more longitude-dependent structures including even some reversed flux patches. 275 Overall, all models tend to agree in their large-scale structure (Table 1) and differ in their 276 quantification of small scale features. 277

All models (1 - 4) show axial quadrupole-dipole ratios of 0.28-0.31 (Table 1) which are significantly smaller than those obtained by dipole offset models (Anderson et al., 2012; Johnson et al., 2012) but are in agreement with a model constructed using spherical caps (Thébault et al., 2018). We note that we could force the models to have a larger quadrupole-dipole ratio close to $g_2^0/g_1^0 = 0.4$ (found by Anderson et al. (2012); Johnson et al. (2012)); however, this leads to 10 - 20% larger rms misfits, which we consider to be significant and eventually deleterious for a large quadrupole-dipole ratio.

Maps in Figure 2 have also structures where no data are available, i.e. in the southern hemisphere. The magnetic field morphology in this hemisphere is mostly determined

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Figure 2. Radial component of the magnetic field at Mercury's core surface of the selected model solutions: (a) Model 1, (b) Model 2, (c) Model 3 and (d) Model 4.

by the global characteristic of the spherical harmonic analysis and the prior used in the inversion. We, therefore do not attempt to interpret magnetic field features in the southern hemisphere.

Power spectra (Lowes, 1966; Mauersberger, 1956) of the models at the core sur-290 face are shown in Figure 3. The spectra mostly match for spherical harmonic degrees 291 1 to 3. We find three different types of spectral slopes for spherical harmonic degrees l > l292 5: decreasing, increasing and flat. The spectral power of Model 3 drops exponentially, 293 whereas Models 1 and 2 show powers that increase by one order of magnitude. Model 294 4 shows a flatter spectrum. This may lead to similar conclusions as taken from the res-295 olution analysis, where for Models 1, 2 and 4 spherical harmonic degrees above l > 8296 may be influenced by spectral leakage of magnetic small-scale sources close to the sur-297 face of Mercury. 298

Common characteristics of all models are represented by their median of the models. This technique is commonly used in the derivation of the International Geomagnetic Reference Field model (IGRF), where a wide variety of geomagnetic main field models based on diverse modeling philosophies are averaged (Thébault, Finlay, Alken, et al., 2015). The discussion of the spatial characteristic of Mercury's time-averaged magnetic field will

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Figure 3. Power spectra of the magnetic field models at Mercury's core surface.

therefore be based on this median model which neither should be too damped (as Model 3) nor too contaminated by spectral leakage at high spherical harmonic degrees (as probably Models 1 and 2). As mentioned earlier (see discussion of Figure 1), we are not convinced by the robustness of spherical harmonic degrees 9 & 10, and therefore we truncate the median model to spherical harmonic degree 8.

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4.2 Mercury's time-averaged magnetic field

Maps in Figure 4 are derived from the median model until spherical harmonic de-310 gree 8. Figures 4a and b show the non-dipole field and the non-axisymmetric, i.e. non-311 zonal, field at Mercury's core surface, respectively. The mapping of the non-dipole field 312 excludes the dipole coefficients g_1^0, g_1^1 and h_1^1 , whereas the non-axisymmetric field excludes 313 all zonal terms, i.e. g_l^0 . The non-dipole field (Figure 4a) is dominated by the equatorial-314 symmetric g_2^0 term. Though the estimate of g_2^0 is strongly influenced by the uneven dis-315 tribution of MESSENGER data, it is the second strongest coefficient of the field. In ad-316 dition, the field features at the northern hemisphere are stronger due to significant equatorial-317 antisymmetric terms, i.e. g_3^0 . Finally, the signature of two normal polarity flux patches 318 at northern high latitudes is evident in Figure 4b. Recall that Figure 4b shows the non-319 zonal field; the two negative structures correspond to two normal polarity flux patches, 320 while the two positive non-zonal structures are the lows in between (see Figure 4a). 321

The polar view map of the non-dipole radial field at the surface of Mercury's core (Figure 4c) shows an elongated patch of intense magnetic flux over the North pole. This



Figure 4. Radial component of: (a) the non-dipole and (b) the non-axisymmetric (non-zonal) magnetic field at Mercury's core surface, respectively. (c) and (d) show north polar views of these magnetic field parts. Maps are derived using the median model truncated at spherical harmonic degree $L_{int} = 8$. The black lines mark the zero contour. Note different scales for maps (a), (c) and (b), (d).

pattern is surrounded by a region of positive (i.e. opposite polarity) magnetic flux with some intensified patterns, where the boundary between these regions of opposite polarity, i.e. the magnetic equator (black line) shows considerable undulations. These undulations further indicate non-axisymmetric field contributions.

The non-zonal field, shown in Figures 4b and d, is fainter. Its amplitude ranges between \pm 200 nT, corresponding to ~ 15% of the total field. This part of the field allows to unravel longitude-dependent structures that are otherwise masked by the strong axisymmetric field. Particularly, the non-zonal field shows four features with alternating signs at high latitudes (A, B, C & D in Figure 5), indicative of two intense normal flux patches. The centers of these flux patches appear approximately at 65° northern latitude.

Furthermore, Mercury's core field also shows non-zonal structures at lower latitudes. These primed features (A', B', C' & D' in Figure 5) are weaker than their higher latitude counterparts. The primed features seem to be shifted relatively to the higher latitude structures by a longitudinal angle of 30° to 60° to the west. The very existence of the primed features may provide further interpretations of processes and structures within Mercury's core. However, their relative weakness might render these interpretations as too speculative.

The primed and un-primed features are not related to magnetic signatures of Birke-342 land currents that are flowing in the north polar region for several reasons. First, because 343 the magnetic field of Birkeland currents is most prominent in the horizontal directions, 344 i.e. it is and absent in the radial component; Birkeland currents flow in radial direction, 345 consequently their induced magnetic field is horizontal. Secondly, their signatures would 346 occur as an auroral band in a time-averaged analysis. The auroral band has a zonal struc-347 ture which is removed from maps of the non-zonal B_r , i.e. in Figures 4b, 4d and 5. Fi-348 nally, the amplitude of features A, B, C & D is about 5 times larger than the magnetic 349 signal of Birkeland currents. 350

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4.3 Inferring Mercury's internal structure

The concentric arrangement of non-zonal features (A, B, C & D) in the northern hemisphere as seen in Figure 5 could be indicative of processes that are involved in the magnetic field generation. To reach conclusions about these processes, we assume that



Figure 5. Polar view of the non-axisymmetric radial magnetic field at Mercury's core surface. The red line shows the position of the magnetic equator, capital letters mark apparent non-zonal field features and pluses their centers. Right: a schematic illustration of the individual convective rolls associated with the high-latitude non-zonal field features (clockwise A, C; counterclockwise B, D).

the nature of these features are linked to columnar rolls tangent to the inner core, par-355 allel to the planet's spin axis i.e. these columnar rolls are expected to be equatorially 356 symmetric. Busse (1975) showed that such columnar flow exists when the Coriolis force 357 dominates viscous and Lorentz forces in the convective region. Oppositely rotating con-358 vective rolls (clockwise and anticlockwise, see Figure 1 of Busse, 1975) may explain the 359 different signs of the non-zonal magnetic field structures. Cyclones/anti-cyclones in the 360 northern hemisphere correlate with convergence/divergence and concentrated/dispersed 361 field, respectively (e.g. Olson et al., 1999)). Accordingly, in Figure 5 the flux patches B 362 & D may be concentrated by fluid downwellings associated with cyclones, while the pos-363 itive non-zonal field (i.e. relatively weak field) in A & C may be dispersed by fluid up-364 wellings associated with anti-cyclones. 365

We interpret the latitude of these flux patches by comparing them to Earth's mag-366 netic core field. Amit et al. (2011) quantitatively identified centers of geomagnetic in-367 tense flux patches. Their Figure 9 and our Figure A.1 suggest that patch centers appear 368 persistently at latitudes somewhat lower than that of the tangent cylinder. Analysis of 369 the qufm1 historical geomagnetic field model (Jackson et al., 2000) reveals that patch 370 latitudes are time-dependent, appearing from about 30° latitude lower than the tangent 371 cylinder until very close to it. However, in recent epochs when the field models are more 372 reliable the patches reside less than $\sim 10^{\circ}$ lower than the tangent cylinder (Amit et al., 373 2011). This agrees with our analysis of a recent IGRF model (Thébault, Finlay, Beggan, 374 et al., 2015). The latitude of the geomagnetic flux patches based on Figure A.1 is ap-375 proximately 8° lower than that of the tangent cylinder. We conclude that based on the 376 behavior of the geomagnetic field the offset between the patches and the actual latitude 377 where the tangent cylinder intersects the CMB is roughly $\delta\theta \sim 10^{\circ} \pm 10^{\circ}$. 378

379 380

Figure 6 (top) illustrates the classical tangent cylinder geometry with the addition of the effect of $\delta\theta$. This geometry is written as

$$\sin(\theta_{cmb} - \delta\theta) = \frac{r_i}{r_o},\tag{6}$$

where r_i and r_o are the radii of the inner core and the CMB respectively. We assume that θ_{cmb} can be obtained from Mercury's non-zonal field and that $\delta\theta$ is similar to Earth's value. This allows to derive r_i , Mercury's inner core size. However, the presence of a stratified layer at the top of the core complicates this inference. When such a layer exists, the convective rolls concentrate flux at the base of the stratified layer, from which a skin ef-



Figure 6. Schematic illustrations of the geometry of the tangent cylinder effect without (top) and with (bottom) a stably stratified layer. See text for the definitions of different angles.

fect may carry the signal to the CMB by diffusion (Christensen, 2006; Christensen & Wicht, 2008). The presence of zonal flows in the stratified layer (Olson et al., 2018; Christensen, 2018) might complicate our inference of the core structure, though these flows would advect the magnetic flux patches in the east-west direction with a lesser impact on their latitudes which is the focus of our analysis.

Assuming that the signal propagation across the stratified layer is roughly radial, then Figure 6 (bottom) illustrates the relation between the co-latitude of the patches θ_{cmb} , the radius of the inner core r_i and the radius of the base of the stratified layer r_s . Mathematically this relation is

$$\sin(\theta_{cmb} - \delta\theta) = \frac{r_i}{r_s},\tag{7}$$

which contains two unknowns, r_i and r_s , and therefore cannot be uniquely determined. However, it provides a useful constraint and may be used to highlight various plausible scenarios for Mercury's internal structure.

To estimate Mercury's inner core size, we first estimate θ_{cmb} from Mercury's mag-398 netic field model. The centers of intense flux patches B & D (Figure 5) reside at about 399 latitude ~65° North, or co-latitude $\theta_{cmb} = 25^{\circ}$. From the analysis of the geomagnetic 400 field we further assume $\delta\theta = 10^{\circ} \pm 10^{\circ}$. Substituting these values into (7) gives sce-401 narios for Mercury's internal structure. Figure 7a presents the results for Mercury's in-402 ner core size r_i and the thickness of the convective region is given in Figure 7b. Both are 403 functions of the radius of the base of the stratified layer r_s , for three values of $\delta\theta$ which 404 cover the considered range. Small values of r_s , which correspond to a very deep base of 405 the layer (thick layer), give a very small inner core which would render the production 406 of non-zonal features at the base of the stratified layer and hence the identification of 407 a tangent cylinder effect to be impossible. Moreover, the magnetic Reynolds number scales 408 with the convective shell thickness; if most of the core is stratified, a dynamo action is 409 unlikely. Larger values of r_s , which correspond to a thinner stratified layer, give a thicker 410 inner core with stronger dependence on $\delta\theta$. For a thin stratified layer of ~50 km, with 411 $\delta\theta = 0^{\circ}$ we find an upper bound $r_i \sim 850$ km and a convective region of 1160 km, with 412 $\delta\theta = 10^{\circ}$ we find $r_i \sim 500$ km and a convective region of 1490 km, whereas with $\delta\theta =$ 413 20° r_i is ~180 km and a convective region of ~1830 km. On the other hand, with a thick 414 stratified layer of ~ 1600 km for all $\delta\theta$ the inner core size is smaller than 200 km and the 415 respective sizes of the active dynamo region are less than 400 km. 416



Figure 7. Mercury's inner core size r_i (a) and the thickness of the convective region (b) vs. the radius of the base of the stratified layer r_s (or the layer thickness, see top horizontal scale), for three values of $\delta\theta$ (see legend).

In the discussion section we elaborate on the consequences of the latitude of the
 magnetic flux patches on inferring the internal structure of Mercury.

419 5 Discussion

Figure 1 can directly be compared to results of a resolution analysis by Uno et al. (2009). The resolution of our preferred model is certainly higher than the resolution of their inversion results. This is mainly because of the wider spatial coverage during the MESSENGER main mission (2011 - 2015) than during the three flybys of Mariner-10 and the one MESSENGER flyby in 2008. However, Uno et al. (2009) concluded that a realistic resolution up to spherical harmonic degree 10 can be obtained from the flyby data. We consider this as an optimistic view, as it (implicitly) assumes that model parameters with a small but non-zero resolution can be resolved by the inversion. In this respect, we are cautious in choosing a minimum resolution R_{min} which would be relevant for robust results. We find that a value of $R_{min} \ge 0.4$ is a diligent choice for a required minimum resolution. The model solutions 1, 2 and 4 largely meet this criterion, and maps of these models are in good agreement when truncated to spherical harmonic degree l = 8. Small-scale structures along the magnetic equator, particularly those seen in Models 1 and 2, arise from spherical harmonic degrees l > 8 which we consider as

uncertain.

434

The tangent cylinder effect is expected to hold when the dynamics is dominated 435 by rapid rotation effects. This is the case in Earth's core (e.g Jault, 2008; Aubert, 2013; 436 Long et al., 2020). However, the dynamical regime in Mercury's liquid core is largely un-437 certain due to the unknown convection vigor there. To reproduce the magnetic equator 438 offset of Mercury's field, the dynamo models of Cao et al. (2014) exhibit a superposition 439 of two unstable columnar convection modes in rapidly rotating spheres, whereas the mod-440 els of Takahashi et al. (2019) contain an anti-symmetric flow component. Overall, cau-441 tion is required when considering our results which would be valid only if rapid rotation 442 effects govern Mercury's core dynamics. Bearing this in mind, our inference of the in-443 ner core size could provide insights to characterize the planet's internal structure and 444 the dynamo action in its core. The morphology of Mercury's non-axisymmetric magnetic 445 field that is shown in Figures 4b and d exhibits two high-latitude normal flux patches. 446 The axisymmetric and non-axisymmetric parts of Mercury's magnetic field may be due 447 to different processes. In this context it has been proposed that a stratified layer out-448 side the dynamo region of Saturn leads to the axisymmetrization of its magnetic field 449 (Stevenson, 1982; Stanley, 2010). Similar mechanisms are likely to be at work inside Mer-450 cury (Christensen, 2006; Christensen & Wicht, 2008). 451

Mercury's internal structure is still unresolved by geodetic analyses and there is a debate concerning the existence and possible size of a solid inner core. If a solid inner core exists, it was argued that its radius is likely to be smaller than ~1000 km (Van Hoolst et al., 2012; Cao et al., 2014; Dumberry & Rivoldini, 2015; Peale et al., 2016). Based on estimates of Mercury's gravity field, tidal Love number and pole coordinates, Mercury's inner core radius is in the range 883 to 1026 km (Genova et al., 2019). However, reported values of Mercury's inner core size are still under debate and estimates derived from a

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geodetic analysis of Mercury's orbital motion give a larger range of 370-1200 km (from
combining the first and third quartiles of Margot et al., 2018).

Based on the above estimates from geodetic analyses we consider Mercury's inner 461 core size to be $r_i = 500{\text{-}}1000$ km. For $r_i = 500$ km and $\delta\theta = 0^{\circ}$ Figure 7(a) gives a strat-462 ified layer thickness of ~ 880 km which leaves ~ 680 km for the convective region to main-463 tain a dynamo. For $r_i = 500$ km and $\delta\theta = 10^{\circ}$ the stratified layer thickness is ~130 464 km and the convective region is ~1430 km, while for $r_i = 500$ km and $\delta\theta = 20^{\circ}$ a so-465 lution does not exist. Furthermore, an inner core size of $r_i = 1000$ km is out of range 466 for the considered $\delta\theta$ values (Figure 7(a)). Because the large scale field of Mercury fa-467 vors a substantial stratified layer, and because large r_i constraints $\delta\theta$ to small admissi-468 ble values, we conclude that the inner core size tends towards the small end of the con-469 sidered r_i range. 470

The thickness of the stratified layer at the top of Mercury's core is also unknown. 471 Smith et al. (2012) suggested that a 200 km thick and solid FeS-layer at the interface 472 of a silicate mantle and the metallic core may explain the planet's moment of inertia. 473 However, this setup was questioned by Hauck et al. (2013) who derived models without 474 an FeS-layer to reproduce the gravity field observations and libration data. The thick-475 ness of such a layer depends on the available Sulfur and its solubility in the metallic core 476 determined by the widely unknown core temperature and reduction conditions (Hauck 477 et al., 2013). In most numerical dynamo simulations that attempt to explain observa-478 tions of Mercury's magnetic field a thick layer is assumed, from several hundred km (e.g. 479 600 km in Christensen, 2006; Christensen & Wicht, 2008) up to half the core radius (Taka-480 hashi et al., 2019). The stratified layer weakens and diffuses the non-axisymmetric field 481 via a skin effect, which could explain its low intensity and dominant axisymmetry. 482

⁴⁸³ Considering a stratified layer thickness of 500-1000 km (or $r_s = 1560-1060$ km), we ⁴⁸⁴ obtain $r_i \sim 660-90$ km and a dynamo region of $\sim 1420-610$ km, respectively with ranges ⁴⁸⁵ corresponding to the different $\delta\theta$ values (Figure 7). The small inner core scenario (with ⁴⁸⁶ $\delta\theta = 20^{\circ}$) seems unlikely to produce a detectable tangent cylinder effect. We therefore ⁴⁸⁷ favor again the solutions for low $\delta\theta$ which correspond here to inner core sizes of $\sim 660-$ ⁴⁸⁸ 450 km and convective region sizes of approximately 900-610 km.

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6 Conclusion 489

In this study, we investigate the morphology of Mercury's magnetic core field and 490 the smallest possible spatial scales that can be resolved from the MESSENGER mea-491 surements. Our spherical harmonic analysis demonstrates that features of the time-averaged 492 magnetic core field of spherical harmonic degree l = 8 can be robustly resolved, inde-493 pendent of the model prior. Higher spherical harmonic degrees are likely aliased by un-494 determined magnetic signatures. Moreover, we detect non-axisymmetric features of the 495 core magnetic field that are absent in the dipole offset model (Anderson et al., 2012; John-496 son et al., 2012). 497

For the first time, Mercury's non-axisymmetric core field is identified and studied 498 to infer the internal structure of its core. We find non-axisymmetric flux patches at high 499 northern latitudes. We interpret these features as the signature of convective columns 500 adjacent to the inner core tangent cylinder. The deviation from axisymmetry introduced 501 by these patches is far less pronounced than at Earth's geomagnetic field due to the mask-502 ing by Mercury's dominant axisymmetric field. 503

We take advantage of the mean latitude of these two patches to constrain Mercury's 504 internal structure. We establish a relation between the inner core size and the thickness 505 of the stratified layer below the CMB as a function of the latitude of the magnetic flux 506 patches. While various combinations of these two quantities are possible, a combined in-507 terpretation of our results and those from geodetic analyses limits the range of the in-508 ner core radius to \sim 500-660 km. Accordingly the stratified layer thickness is \sim 880-500 509 km, leaving $\sim 900-610$ km for the convective dynamo region, respectively. Furthermore, 510 our results favor little (if any) shift between the locations of magnetic flux patches and 511 the tangent cylinder at the top of the dynamo region, in apparent contrast to the off-512 set observed at Earth's core. 513

514

Finally we emphasize that our analysis is based on a data set of the MESSENGER 515 mission over the northern hemisphere only. This puts limits on the magnetic field models and the inferences concerning Mercury's internal structure. The future Bepi-Colombo 516 mission will unravel these details of Mercury's magnetic core field. 517

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531

The authors declare that they have no competing interests.

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Figure A.1. North polar views of the radial magnetic field component (a) and its nonaxisymetric part (b) at Earth's core surface. The red circles represent the diameter of Earth's inner core.

718 A Earth's core field

Figure A.1 shows the radial geomagnetic field component and its non-axisymetric part at Earth's core surface in the year 2015. The maps are based on the 12^{th} International geomagnetic reference field (Thébault, Finlay, Beggan, et al., 2015). The model was truncated at spherical harmonic degree $L_{\text{int}} = 10$. The maps show also the projection of the inner core tangent cylinder on the CMB.

⁷²⁴ B Covariance analysis

We study the robustness of our inversion results by analyzing the resolution matrix. Yet, another way to estimate formal uncertainties of the results is by analyzing the covariance matrix, which is given by

$$\mathbf{C} = \hat{\sigma}^2 (\mathbf{A}^{\mathsf{T}} \mathbf{C}_{\mathbf{e}}^{-1} \mathbf{A} + \lambda_s \mathbf{C}_{\mathbf{m}}^{-1})^{-1}, \qquad (B.1)$$

where $\hat{\sigma}^2$ is the misfit between model and data. These errors are formal, as they represent the uncertainty in the model subject with respect to the constraint and may be invalidated by false observations or by inappropriate prior information. It does not contain that part of the uncertainty, which is related to a trade-off in resolution, when combinations of parameters have the same effect on the fit to the data (Bloxham et al., 1989).



Figure B.1. Diagonal terms of the four models covariance matrices.

In this sense, diagonal elements of C are the formal variances of the model parameters,
and the off-diagonal elements are the formal covariance between individual model parameters.

In Figure B.1 the diagonal elements of the four models covariance matrices are shown. The largest variances are found for coefficients of spherical harmonic degrees 2 and 3, i.e. g_2^1 and g_3^1 . Generally, the formal error is small, and depends on the strength of the damping parameter λ_s ; weaker damping enhances the formal error.

Figure B.2 shows the scaled covariance matrix elements of the four models. These 740 matrix elements are scaled by the variances of the diagonal elements. This scaling pro-741 cedure enhances covariance structures of the non-diagonal elements. The scaled covari-742 ance matrices of Models 1, 2, and 4 are very similar in their off-diagonal structures, whereas 743 those structures are rather faint in the covariance matrix of Model 3. Large negative co-744 variance (blue) occur between g_2^0 and coefficients of the first spherical harmonic degree 745 (lower left corner of each plot). This trend continues between coefficients of consecutive 746 spherical harmonic degrees e.g. $C(g_3^1, g_4^1)$, which leads to 'parallel' off-diagonal structures. 747 Positive covariance structures are less pronounced and occur between coefficients with 748 two spherical harmonic degrees differences e.g. $C(g_2^0, g_4^0)$. These structures are mostly 749 visible for Models 1 and 2. The cause of these off-diagonal structures is the uneven hemi-750 spherical data distribution. 751



Figure B.2. Covariance matrix elements of the four model solutions. Elements are scaled by the covariance of the diagonal elements $C(g_n^n, g_n^n)$ to enhance the visibility of the non-diagonal terms. a, b, c and d refer to matrices of Models 1, 2, 3 and 4, respectively.