Bath's law derived from magnitude distribution

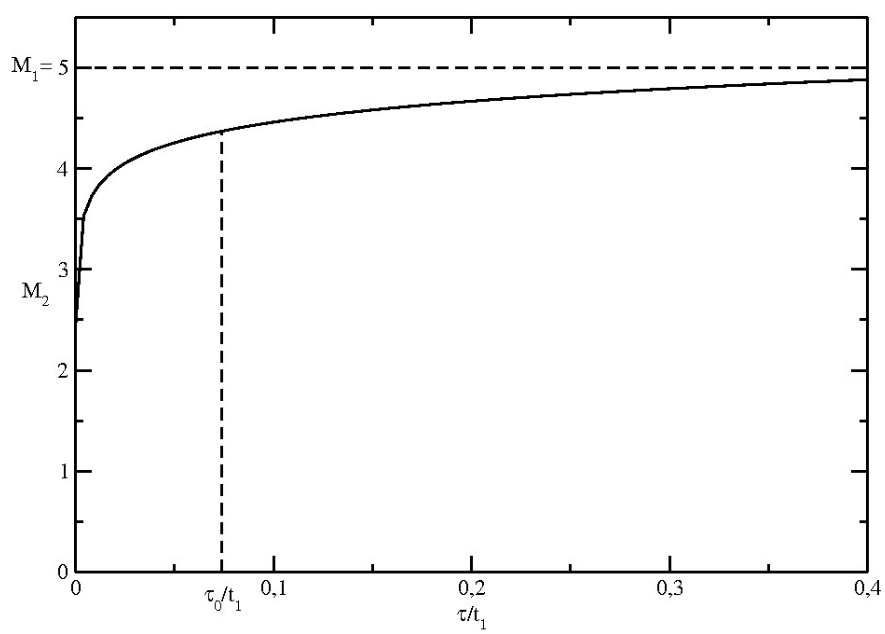
Bogdan Felix Apostol[1]

[1]Institute for Earth'sc Physics, Magurele-Bucharest

November 23, 2022

Abstract

The empirical Bath's law is derived from the statistical Gutenberg-Richter distribution in magnitude difference of pairs of earthquakes. The derivation of the statistical Gutenberg-Richter distributions in energy and magnitude is presented, as resulting from a geometric-growth model of energy accumulation in the focal region. It is shown that the most suitable framework of understanding the origin of the Bath's law is the extension of the statistical distributions to pairs of earthquakes, where the difference in magnitude is allowed to take negative values. If the seismic activity which accompanies a main shock is viewed as a relaxation process, then we need to include both the aftershocks and the foreshocks in this accompanying seismic activity, and to view it as fluctuations in magnitude. The extension of the magnitude difference to negative values leads to a vanishing mean value of the fluctuations and to accepting the standard deviation as a measure of these fluctuations. It is suggested that the standard deviation of the magnitude difference is the average difference in magnitude between the main shock and its largest aftershock (foreshock), thus providing an insight into the nature and the origin of the Bath's law. The geometric-growth model of energy accumulation in the focal region induces a lower bound to the magnitudes of the largest aftershocks (foreshocks), such that the (average) reference value $\Delta M=1.2$ between the magnitudes of the main shock and the largest accompanying seismic event corresponds to the smallest aftershock (foreshock) in the whole set of the largest aftershocks (foreshocks).



Bath's law derived from magnitude distribution

B. F. Apostol
Department of Seismology,
Institute of Earth's Physics, Magurele-Bucharest MG-6,
POBox MG-35, Romania
email: afelix@theory.nipne.ro

Abstract

The empirical Bath's law is derived from the statistical Gutenberg-Richter distribution in magnitude difference of pairs of earthquakes. The derivation of the statistical Gutenberg-Richter distributions in energy and magnitude is presented, as resulting from a geometric-growth model of energy accumulation in the focal region. It is shown that the most suitable framework of understanding the origin of the Bath's law is the extension of the statistical distributions to pairs of earthquakes, where the difference in magnitude is allowed to take negative values. If the seismic activity which accompanies a main shock is viewed as a relaxation process, then we need to include both the aftershocks and the foreshocks in this accompanying seismic activity, and to view it as fluctuations in magnitude. The extension of the magnitude difference to negative values leads to a vanishing mean value of the fluctuations and to accepting the standard deviation as a measure of these fluctuations. It is suggested that the standard deviation of the magnitude difference is the average difference in magnitude between the main shock and its largest aftershock (foreshock), thus providing an insight into the nature and the origin of the Bath's law. The geometric-growth model of energy accumulation in the focal region induces a lower bound to the magnitudes of the largest aftershocks (foreshocks), such that the (average) reference value $\Delta M = 1.2$ between the magnitudes of the main shock and the largest accompanying seismic event corresponds

to the smallest aftershock (foreshock) in the whole set of the largest aftershocks (foreshocks).

1 Introduction

Bath's law states that the average difference ΔM between the magnitude of a main shock and the magnitude of its largest aftershock is a number independent of the magnitude of the main shock (Bath, 1965; see also Richter, 1958). The reference value of the average magnitude difference is $\Delta M = 1.2$. Deviations from this value have been reported (see, for instance, Felzer et al, 2002; Console et al, 2003), some being discussed even by Bath, 1965.

The Bath's law is an empirical law. The earliest advance in understanding its origin was made by Vere-Jones, (1969), who viewed the main shock and its aftershocks as statistical events of the same statistical ensemble, distributed in magnitude. The distribution in magnitude difference, introduced by Vere-Jones, 1969, implies correlations, which are viewed sometimes as reflecting the opinion that the main shocks are statistically distinct from the aftershocks (Utsu, 1969. Evison and Rhoades, 2001). The Bath's law enjoyed many discussions and attempts of elucidation (Papazachos, 1974; Purcaru, 1974; Tsapanos, 1990; Kisslinger and Jones, 1991; Evison, 1999; Lavenda and Cipollone, 2300; Lombardi, 2002; Helmstetter and Sornette, 2003). The prevailing opinion ascribes the variations in ΔM to the bias in selecting data and the insufficiency of the realizations of the statistical ensemble. This standpoint was substantiated by means of the binomial distribution for the earthquakes (Console, 2003; Lombardi, 2002; Helmstetter and Sornette, 2003). In order to account for the deviations of ΔM Helmstetter and Sornette, 2003, employed the ETAS model (epidemic-type aftershock sequence) for the differences in the selection procedure of the mainshocks and the aftershocks. These authors showed that the variations in the number ΔM are related to the realizations of the statistical ensemble and the values of the fitting parameters (see also Lombardi, 2002; Console, 2003).

The statistical hypothesis for the distributions of earthquakes is far-reaching. Usually, a statistical distribution is independent of time (it is an equilibrium distribution), such that the aftershocks distribution should be identical with the foreshocks distribution. However, it seems that there are differences between these two distributions (Utsu, 2002; Shearer, 2012). Then, for the law to be operational in practice, we need to have a large statistical ensemble of aftershocks (and foreshocks), disentangled from the main shock, which is a difficult issue. In addition, the realizations of the distribution should be repetitive, a point which is debatable in different conditions of moments of time and locations.

We show in this paper that the appropriate tool of discussing the accompanying seismic activity (foreshocks and aftershocks) of the main shocks is the pair distribution function for the difference in magnitude, according to Vere-Jones, 1969. The derivation of this distribution is made herein by means of the conditional probabilities and the Bayes theorem. The difference in magnitude is extended to the whole real axis, leading to a symmetric distribution for the foreshocks and aftershocks, with a vanishing mean value for the magnitude difference. This suggests to view the accompanying seismic activity as consisting of fluctuations, and to take their standard deviation as a measure for the Bath's average difference ΔM between the magnitude of the main shock and its largest aftershock (foreshock). Moreover, we show here that the reference value $\Delta M = 1.2$ corresponds to the smallest aftershock (foreshock) in the whole set of the largest aftershocks (foreshocks), *i.e.* it is a lower bound to the (average) magnitude of the largest accompanying seismic events. This later point is related to the geometry of energy-accumulation process in the focal region. By pointing out a limiting value, the Bath's law acquires, indeed, a special relevance.

2 Gutenberg-Richter statistical distributions

Apostol, 2006a,b, put forward a geometric-growth model of energy accumulation in a localized earthquake focal region. According to this model, the accumulated energy E is related to the accumulation time t by

$$1 + t/t_0 = (1 + E/E_0)^r \quad , \tag{1}$$

where t_0 and E_0 are time and energy thresholds and r is a geometrical parameter which characterizes the focal region. This parameter is related to the reciprocal of the number of effective dimensions of the focal region and to the strain accumulation rate (which, in general, is anisotropic). Very likely, the parameter r varies in the range $1/3 < r < 1$. For a pointlike focal region with a uniform accumulation rate $r = 1/3$ (three dimensions), for a two-dimensional uniform focal region $r = 1/2$, while for a one-dimensional focal region r tends to unity. An average parameter r may take any value in this range.

In equation (1) the threshold parameters should be viewed as very small, such that t/t_0, $E/E_0 \gg 1$ and equation (1) may be written as

$$t/t_0 \simeq (E/E_0)^r \quad . \tag{2}$$

A uniform frequency of events $\sim t_0/t$ in time t indicates that the parameter t_0 may be viewed as the reciprocal of a seismicity rate $1/t_0$. It follows immediately the time distribution

$$P(t)dt = \frac{1}{(t/t_0)^2}\frac{dt}{t_0} \tag{3}$$

and, making use of equation (2), the energy distribution

$$P(E)dE = \frac{r}{(E/E_0)^{1+r}}\frac{dE}{E_0} \quad . \tag{4}$$

At this point we may use an exponential law $E/E_0 = e^{bM}$, where M is the earthquake magnitude and $b = \frac{3}{2} \cdot \ln 10 = 3.45$, according to Kanamori, 1977, and Hanks and Kanamori, 1979 (see also, Gutenberg and Richter, 1944, 1956); we get the (normalized) magnitude distribution

$$P(M)dM = \beta e^{-\beta M} dM \quad , \tag{5}$$

where $\beta = br$. In decimal logarithms, $P(M) = \beta \cdot 10^{1.5r}$, where $0.5 < 1.5r < 1.5$ (for $1/3 < r < 1$). Usually, the average value $1.5r = 1$ ($\beta = 2.3$) is currently used as a reference value, corresponding to $r = 2/3$ ($\simeq 0.66$) (see, for instance, Stein and Wysesssion, 2003; Udias, 1999; Lay and Wallace, 1995; Frohlich and Davis, 1993).

It is worth noting that the magnitude distribution (equation (5)) has the property $P(M_1 + M_2) \sim P(M_1)P(M_2)$, while the time and energy distributions (equations (3) and (4)) have not this property. This is viewed sometimes as indicating that the earthquakes would be correlated in time of occurrence, and in energy, but not in magnitude (Corral, 2006).

The magnitude distribution is particularly important because it can be used to analyze the empirical distribution

$$P(M) = \frac{\Delta N}{N_0 \Delta M} = \frac{t_0 \Delta N}{T \Delta M} \ , \tag{6}$$

of ΔN earthquakes with magnitude in the range $(M,\, M + \Delta M)$ out of a total number $N_0 = T/t_0$ of earthquakes occurred in time T. We get

$$\ln(\Delta N/T) = \ln\left(\frac{\beta \Delta M}{t_0}\right) - \beta M \ . \tag{7}$$

From the magnitude frequency $\Delta N/T$ (equation (6)) we get the mean recurrence time

$$t_r = \frac{t_0}{\beta \Delta M} e^{\beta M} \tag{8}$$

for an earthquake with magnitude M (*i.e.* in the interval $(M,\, M + \Delta M)$). This time should be compared with the accumulation time $t_a = t_0 e^{\beta M}$ for an earthquake with magnitude M, given by equation (2) and the exponential law $E/E_0 = e^{bM}$. These times are related by $t_a = (\beta \Delta M) t_r$, whence one can see that $t_a < t_r$ (for $\beta \Delta M < 1$), a relationship which shows that the energy corresponding to a magnitude M may be lost by seismic events lower in magnitude, as expected. Moreover, by the definition of the seismicity rate, an earthquake with magnitude M is equivalent with a total number $t_a/t_0 = e^{\beta M}$ of earthquakes with zero magnitude (energy E_0) (Felzer et al, 2002; Michael and Jones, 1998). It is worth noting that the magnitude distribution $\beta e^{-\beta M}$ implies an error of the order $\left(\sqrt{\overline{M^2}} - \overline{M}\right)/\overline{M} = \sqrt{2} - 1$ at least, *i.e.*, $\Delta t_r / t_r \simeq 0.41$, which is too large to be useful. For a maximal entropy with mean recurrence time t_r we get easily a Poisson distribution $(1/t_r) e^{-t/t_r}$ for the recurrence time, which has a large standard deviation $\sqrt{\overline{(t - t_r)^2}} = t_r$.

Similarly, from equation (5) we get the excedence rate (the so-called recurrence law), which gives the number N_{ex} of earthquakes with magnitude greater than M. The corresponding probability is readily obtained from (5) as $P_{ex} = e^{-\beta M}$, such that the excedence rate reads

$$\ln N_{ex} = \ln N_0 - \beta M \ . \tag{9}$$

The distributions given above may be called the statistical Gutenberg-Richter distributions (equations (7) and (9) and, implicitly, (5)). They are currently used in statistical analysis of the earthquakes. The parameter β is derived by fitting these distributions to data. For $1/3 < r < 1$ it varies in the range $1.15 < \beta < 3.45$ (for decimal logarithms $0.5 < 1.5r < 1.5$). For instance, an analysis of a large set of global earthquakes with $5.8 < M < 7.3$ ($\Delta M = 0.1$) indicates $\beta = 1.38$ (and $1/t_0 = 10^{5.5}$ per year), corresponding to $r = 0.4$, a value which suggests an intermediate two/three-dimensional focal mechanism (Bullen, 1963). For $r = 1/3$, corresponding to a uniform pointlike focal geometry, we get $\beta = 1.15$. Equations (5), (7) and (9) have been fitted to a set of 1999 earthquakes with magnitude $M \geq 3$ ($\Delta M = 0.1$), which occurred in Vrancea between $1974 - 2004$ (31 years) (Apostol 2006a,b). The mean values of the fitting parameters are $-\ln t_0 = 9.68$ and $\beta = 1.89$ ($r = 0.54$). The same fit have been done for a set of 3640 earthquakes with magnitude $M \geq 3$ which occurred in Vrancea during $1981 - 2018$ (38 years). The fitting parameters for this set are $-\ln t_0 = 11.32$ and $\beta = 2.26$ ($r = 0.65$). The data for Vrancea have been taken from the Romanian Earthquake Catalog, 2008).

The statistical analysis gives a generic image of a collective, global earthquake focal region (a distribution of foci). Particularly interesting is the parameter r, which is related to the reciprocal of the (average) number of effective dimensions of the focal region and the rate of energy accumulation. The value $r = 0.54$ (Vrancea, period $1974 - 2004$) indicates a (quasi-) two-dimensional geometry of the focal region in Vrancea, while the more recent value $r = 0.65$ for the same region suggests an evolution of this (average) geometry towards one dimension. At the same time, we note an increase of the seismicity rate $1/t_0$ in the recent period in Vrancea. The increase of the geometrical parameter r determines an increase of the parameter β, which dominates the mean recurrence time. For instance, the accumulation time for magnitude $M = 7$ is increased from $t_a \simeq 34.9$ years (period $1974 - 2004$) to at least $t_a \simeq 59$ years. This large variability indicates the great sensitivity of the statistical analysis to the data set, and, consequently, the limited usefulness of the statistical analyses. In particular, for any fixed M we may view the exponential $Me^{-M\beta}$ as a distribution of the parameter β, which indicates an error $\simeq 0.41$ in determining this parameter.

Also, we note that inherent errors occur in a statistical analysis. For instance, an error is associated to the threshold magnitude $M = 3$, because the large amount of data with $M < 3$ may affect the fit. Also, it is difficult to include events with high magnitude in a set with statistical significance, because such events are rare. In addition, the size of the statistical set may affect the results. The fitting values given above have an error of approximately 10%. Such difficulties are carefully analyzed (*e.g.*, Felzer et al, 2002; Console et al, 2003; Lombardi, 2002; Helmstetter and Sornette, 2003).

The statistical distributions given above may be employed to estimate conditional probabilities, and to derive Omori laws for the associated (accompanying) seismic activity. Also, the conditional probabilities can be used for analyzing the next-earthquake distributions (inter-event time distributions), (Apostol and Cune, 2020) which may offer information for seismic hazard and risk estimation. We present here another example of using these distributions, in analyzing the Bath's empirical law.

3 Bath's law

If the (statistical) Gutenberg-Richter distribution $\sim e^{-\beta M}$ can be viewed as a statistical distribution, it follows that we may view all the earthquakes with magnitude M as members of the same statistical ensemble, characterized by the parameter β. Such a set of earthquakes includes both the main shocks and the events of the accompanying seismic activity, *i.e.* the foreshocks and the aftershocks (Kisslinger, 1996). This is the standpoint of Vere-Jones, 1969. Let us look for the distribution of the difference in magnitude $M_1 - M_2$ of any pair of seismic events with magnitude M_1 and M_2. Looking for this magnitude-difference distribution, we already assume that the two events $M_{1,2}$ are correlated. In the Gutenberg-Richter law the magnitude M is positive, but for the difference $M_1 - M_2$ we need to extend this variable to negative values. Since $M_1 = M_1 - M_2 + M_2$ and $M_2 = M_2 - M_1 + M_1$, the Gutenberg-Richter laws $\sim e^{-\beta M_{1,2}}$ gives a magnitude-difference distribution $\sim e^{-\beta(M_1 - M_2)}$ for $M_1 > M_2$ and fixed M_2, and a distribution $\sim e^{-\beta(M_2 - M_1)}$ for $M_2 > M_1$ and fixed M_1. These are conditional probabilities (related to the Bayes theorem). In both cases, these distributions can be written as $\sim e^{-\beta|m|}$, where $m = M_1 - M_2$ (or $m = M_2 - M_1$), irrespective of which $M_{1,2}$ is fixed. It follows that the (normalized) distribution of the difference in magnitude m is

$$p(m)dm = \frac{1}{2}\beta e^{-\beta|m|}dm \ , -\infty < m < +\infty \ . \tag{10}$$

Making use of this distribution, we can see that the mean value of the magnitude difference is zero, $\overline{m} = 0$ (as expected for fluctuations), while the mean value of the squared difference in magnitude is

$$\overline{m^2} = \frac{2}{\beta^2} \ . \tag{11}$$

It is worth noting that by using the distribution of the pairs given by equation (10), the partners of the pair are not independent of one another anymore; they become correlated, because, for a given value of m the magnitude M_1 (or M_2) cannot take any value, irrespective of the value taken by the magnitude M_2 (or M_1). In spite of their similar (exponential) form, the pair distribution $p(m)$ is different from the distribution $\sim e^{-\beta M}$ of independent magnitudes, because the statistical variable $m = M_1 - M_2$ is different from the statistical variable M ($M = M_{1,2}$).

If the foreshocks may foretell a relaxation of the accumulated seismic stress and the aftershocks are viewed as a relaxation after the main seismic event, then the standard deviation

$$\Delta m = \sqrt{\overline{m^2} - \overline{m}^2} = \left(\overline{m^2}\right)^{1/2} = \frac{\sqrt{2}}{\beta} \tag{12}$$

may be taken as a measure of the largest fluctuation of the statistical equilibrium of the ensemble of magnitude differences. It follows that the quantity given by equation (12) may be viewed as the average difference in magnitude $\Delta M = \Delta m$ between the main shock and its largest aftershock (or foreshock). This is the Bath's law. The number $\sqrt{2}/\beta$ does not depend on the magnitudes $M_{1,2}$ (but it depends on the parameter β, corresponding to various realizations of the statistical ensemble). It is worth noting that Δm given by equation (12) implies an averaging of the squared magnitude differences.

Making use of $\beta = 1.15$ ($r = 1/3$) given above, we get $\Delta m = 1.23$, a value close to the reference value of the Bath's law; for $\beta = 1.38$ ($r = 0.4$) (Bullen, 1963) we get $\Delta m = 1.02$, for $\beta = 1.89(2.26)$ (Vrancea region, Apostol, 2006a,b) we get $\Delta m = 0.75(0.62)$. For the average value $\beta = 2.3$ ($1.5r = 1$)

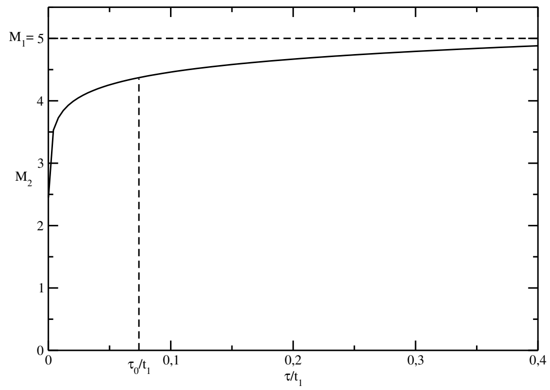


Figure 1: The magnitude M_2 of the accompanying seismic events *vs* the time τ elapsed from the main event with magnitude M_1 and accumulation time t_1 (equation (16) for $M_1 = 5$, $b = 3.45$, $r = 0.66$). The Bath partner $M_2 \simeq 4.38$ corresponds to $\tau_0/t_1 = 0.078$. Higher values of the magnitude M_2 occur at much longer times, where the correlations are unlikely.

we get $\Delta m = 0.61$. We can see that $\Delta m = \Delta M$ is variable, depending on the fitting parameter β, which can be obtained from the statistical analysis of the data; this is in agreement with the prevailing opinion in statistical analysis of the earthquakes (see, for instance, Lombardi, 2002; Console et al, 2003; Helmstetter and Sornette, 2003, and References therein).

According to the geometric-growth model of energy accumulation in the focal region the parameter r varies in the range $1/3 < r < 1$. It entails a variation of the parameter $\beta = \frac{3}{2} \ln 10 \cdot r$ in the range $1.15 < \beta < 3.45$. The parameter r cannot acquire values smaller than $r = 1/3$, because the maximum number of effective dimensions in a focal region cannot be higher than 3 (nor lower than 1). Consequently, the largest value of the Bath's difference ΔM is $\Delta M = 2\sqrt{2}/\ln 10 = 1.23$. We can say that the reference value $\Delta M = 1.2$ of the Bath difference corresponds to its largest value among all possible values in the range $0.41 < \Delta M < 1.23$. This value corresponds to the smallest accompanying seismic event among all the largest ones. It indicates a lower bound to the magnitudes of the largest aftershocks (and foreshocks). By this limiting value, the Bath's law (with $\Delta M = 1.2$) acquires indeed a certain appearance of an absolute law. This result has a formal similarity with the limit theorem of Vere-Jones, 2008.

The correlation between the partners of a pair can be seen form the estimation of the accumulation time t given by equation (2) for a pair of earthquakes with energies $E_{1,2}$, $E = E_1 + E_2$; from equation (2) we get

$$t/t_0 = (E/E_0)^r = (E_1/E_0 + E_2/E_0)^r = \\ = \left(e^{bM_1} + e^{bM_2}\right)^r = (t_1/t_0)\left(1 + e^{-bm}\right)^r \quad , \tag{13}$$

where t_1 is the accumulation time of the earthquake with energy E_1 (magnitude M_1), viewed as the main shock; the earthquake with magnitude $M_2 = M_1 - m$ ($m > 0$) is viewed as the largest aftershock (foreshock). It follows that the occurrence time τ_0 of this "Bath partner" ($m = \Delta m$), measured from the occurrence time of the main shock ($t = t_1 + \tau_0$), is given by

$$\tau_0 = t_1\left[\left(1 + e^{-b\Delta m}\right)^r - 1\right] \simeq \\ \simeq rt_1 e^{-b\Delta m} = rt_1 e^{-\sqrt{2}/r} \tag{14}$$

(for $b\Delta m \gg 1$). We can see that the duration τ_0 depends on the accumulation time t_1 of the main shock, which reflects the correlation between the two events. The ratio τ_0/t_1 varies between 5×10^{-3} ($r = 1/3$) and 0.24 ($r = 1$); for $r = 0.65$ (Vrancea) we get $\tau_0/t_1 = 0.074$.

It is worth noting, according to equation (13), that an associated partner close to the main shock in magnitude ($bm \ll 1$) occurs after a lapse of time

$$\Delta t \simeq t_1\left(2^r - 1\right) \quad , \tag{15}$$

which is greater than τ_0 ($\Delta t/t_1$ varies between 0.26 and 1). We can see that, even if the pair probability $p(m) = (2/\beta)e^{-\beta|m|}$ is greater for $m = 0$, an earthquake close in magnitude to the main shock occurs much later, where it may be difficult to view it as an aftershock (and similarly for the foreshocks). Since $\left(1 + e^{-bm}\right)^r$ is a decreasing function of m, we can say, indeed, that the largest aftershock is farther in time with respect to the main shock in comparison with aftershocks lower in magnitude. The duration τ_0 given by equation (14) for the occurrence of the largest aftershock may be taken as a measure of the extension in time of the aftershock (and the foreshock) activity.

From equation (13) we can get the distribution of the magnitudes M_2 of the acompanying earthquakes with respect to the time τ, measured from the occurrence of the main shock with magnitude M_1, either in the future (aftershocks) or in the past (foreshocks). Indeed, we get from equation (13),

$$M_2 = M_1 + \frac{1}{b} \ln \left[(1 + \tau/t_1)^{1/r} - 1 \right] \ , \tag{16}$$

where t_1 $(= t_0 e^{\beta M_1})$ is the accumulation time of the main shock; M_2 in equation (16) is defined for $\left(1 + e^{-bM_1}\right)^r - 1 < \tau/t_1 < 2^r - 1$ $(0 < M_2 < M_1)$. The function M_2 is plotted in Fig. 1 *vs* τ/t_1 for $b = 3.45$, $r = 2/3$ $(\simeq 0.66)$ and $M_1 = 5$ $(\beta = 2.3)$. For τ/t_1 very close to zero M_2 is vanishing, and for $\tau/t_1 \longrightarrow 2^r - 1$ the magnitude M_2 tends to M_1. The Bath partner occurs at $\tau_0/t_1 \simeq re^{-\sqrt{2}/r} \simeq 0.078$ with the magnitude $M_2 \simeq 4.39$ $(\Delta M \simeq 0.61)$. The function $M_2(\tau/t_1)$ is a very steep function, for the whole (reasonable) range of parameters; the whole accompanying seismic activity is, practically, concentrated in the lapse of time τ_0. On the scale τ/t_1 the pair probability of this activity is an abruptly increasing function of M_2.

Finally, we note that in empirical studies the parameter ΔM is an average over various realizations of statistical ensembles. Such realizations may include focal regions with various parameters r, or variations in time of the parameter r, such that the resulting, effective value of r in the above formulae is larger than 1/3 and, consequently, the effective value of β is larger than 1.15, and the resulting ΔM is smaller than 1.2. Therefore, in such circumstances, the value ΔM remains only a theoretical limit. The results may tend to this value by adjusting the magnitude cutoffs, (Lombardi, 2002; Console, 2003) or by chosing particular values of (many) fitting parameters (Helmstetter and Sornette, 2003); there are cases when the data exhibit values close to $\Delta M = 1.2$ (Felzer et al, 2002).

4 Concluding remarks

The Gutenberg-Richter-type statistical distributions are derived from a geometric-growth model of energy accumulation in the focal region, where a particularly interesting geometrical parameter reflects the (average) geometry of the focal region and the rate of the accumulation process. The statistical distribution in magnitude is used to derive the empirical Bath's law. The main role in this derivation is played by the distribution of the difference in magnitude of earthquake pairs, where the magnitude difference, as a statistical variable, is allowed to extend to negative values. Making use of this pair distribution, the mean value of the magnitude difference is zero and the standard deviation is viewed as a measure of the average difference in magnitude between the main shock and its largest aftershock (foreshock). The associated seismic activity of foreshocks and aftershocks is viewed as consisting of fluctuations in a relaxation process. The use of the pair distribution in magnitude difference is based on the assumption made by Vere-Jones of the same statistical ensemble for both the main shocks and the accompanying foreshocks and aftershocks (Vere-Jones, 1969), though the pair distribution implies correlations. It is shown that the Bath's deviation in magnitude is a statistical parameter, which depends on the particular realization of the statistical ensemble. The reference value $\Delta M = 1.2$ for Bath's difference in magnitude corresponds to a lower bound to the magnitudes of the largest aftershocks and foreshocks.

Acknowledgements

The author is indebted to the colleagues in the Department of Seismology, Institute of Earth's Physics, Magurele-Bucharest, for many enlightening discussions. This work was partially carried out within the Program Nucleu 2016-2018, funded by Romanian Ministry of Research and Innovation, Research Grant #PN16-35-01-07/2016 and #PN18-15-01-01/2018. Data used for the fit (Vrancea region) have been extracted from the Romanian Earthquake Catalog, 2008. Part of them (period 1974-2004) are available through Apostol (2006a,b).

REFERENCES

Apostol, B. F., (2006a). A Model of Seismic Focus and Related Statistical Distributions of Earthquakes. *Phys. Lett.*, A357, 462-466

Apostol, B. F. (2006b). A Model of Seismic Focus and Related Statistical Distributions of Earthquakes. *Roum. Reps. Phys.*, 58, 583-600

Apostol, B. F. & Cune, L. C. (2020). Short-term seismic activity in Vrancea. Inter-event time distributions. *Ann. Geophys.*, to appear

Bath, M. (1965). Lateral inhomogeneities of the upper mantle. *Tectonophysics*, 2, 483-514

Bullen, K. E. (1963). *An Introduction to the Theory of Seismology*. London: Cambridge University Press.

Console, R., Lombardi, A. M., Murru, M. & and Rhoades, D. (2003). Bath's law and the self-similarity of earthquakes. *J. Geophys. Res.*, 108, 2128 10.1029/2001JB001651

Corral, A. (2006). Dependence of earthquake recurrence times and independence of magnitudes on seismicity history. *Tectonophysics*, 424, 177-193

Evison, F. (1999). On the existence of earthquake precursors. Ann. Geofis., 42, 763-770

Evison, F. & and Rhoades, D. (2001). Model of long term seismogenesis. *Ann. Geophys.*, 44, 81-93

Felzer, K. R., Becker, T. W., Abercrombie, R. E., Ekstrom, G. & Rice, J. R. (2002). Triggering of the 1999 M_w 7.1 Hector Mine earthquake by aftershocks of the 1992 M_w 7.3 Landers earthquake., *J. Geophys. Res.*, 107, 2190 10.1029/2001JB000911

Frohich, C. & Davis, S. D. (1993). Teleseismic b values; or much ado about 1.0. *J. Geophys. Res.*, 98, 631-644

Gutenberg, B. & Richter, C. (1956). Magnitude and energy of earthquakes. *Annali di Geofisica*, 9, 1-15 ((2010). *Ann. Geophys.*, 53, 7-12)

Gutenberg, B. & Richter, C. (1944). Frequency of earthquakes in California, *Bull. Seism. Soc. Am.*, 34, 185-188

Hanks, T. C. & Kanamori, H. (1979). A moment magnitude scale. *J. Geophys. Res.*, 84 2348-2350

Helmstetter, A. & Sornnette, D. (2003). Bath's law derived from the Gutenberg-Richter law and from aftershock properties. *Geophsy. Res. Lett.*, 30, 2069 10.1029/2003GL018186

Kanamori, H. (1977). The energy release in earthquakes. *J. Geophys. Res.*, 82, 2981-2987

Kisslinger, C. (1996). Aftershocks and fault-zone properties. *Adv. Geophys.*, 38 1-36

Kisslinger, C. & Jones, L. M. (1991). Properties of aftershock sequences in Southern California. *J. Geophys. Res.*, 96 11947-11958

Lavenda, B. H. & Cipollone, E. (2000). Extreme value statistics and thermodynamics of earthquakes: aftershock sequences. *Ann. Geofis.*, 43 967-982

Lay, T. & Wallace, T. C. (1995). *Modern Global Seismology.* San Diego, CA: Academic.

Lombardi, A. M. (2002). Probability interpretation of "Bath's law. *Ann. Geophys.*, 45 455-472

Michael, A. J. & Jones, L. M. (1998). Seismicity alert probability at Parkfield, California, revisited. *Bull. Seism. Soc. Am.*, 88, 117-130

Papazachos, P. C. (1974). On certain aftershock and foreshock parameters in the area of Greece. *Ann. Geofis.*, 24, 497-515

Purcaru, G. (1974). On the statistical interpretation of the Bath's law and some relations in aftershock statistics. *Geol. Inst. Tech. Ec. Stud. Geophys. Prospect.* (Bucharest), 10, 35-84

Richter, C. F. (1958). *Elementary Seismology* (p.69). San Francisco, CA: Freeman.

Romanian Earthquake Catalogue (ROMPLUS Catalog). (2018). National Institute for Earth Physics, Romania.

Shearer, P. M. (2012). Self-similar earthquake triggering, Bath's law, and foreshock/aftershock magnitudes: simulations, theory, and results for Southerm California. *J. Geophys. Res.*, 117, B06310, 10.1029/2011JB008957

Stein, S. & Wysession, M. (2003). *An Introduction to Seismology, Earthquakes, and Earth Structure.* NY: Blackwell.

Tsapanos, T. M. (1990). Spatial distribution of the difference between magnitudes of the main shock and the largest aftershock in the circum-Pacific belt. *Bull. Seism. Soc. Am.*, 80, 1180-1189

Udias, A. (1999). *Principles of Seismology.* NY: Cambridge University Press.

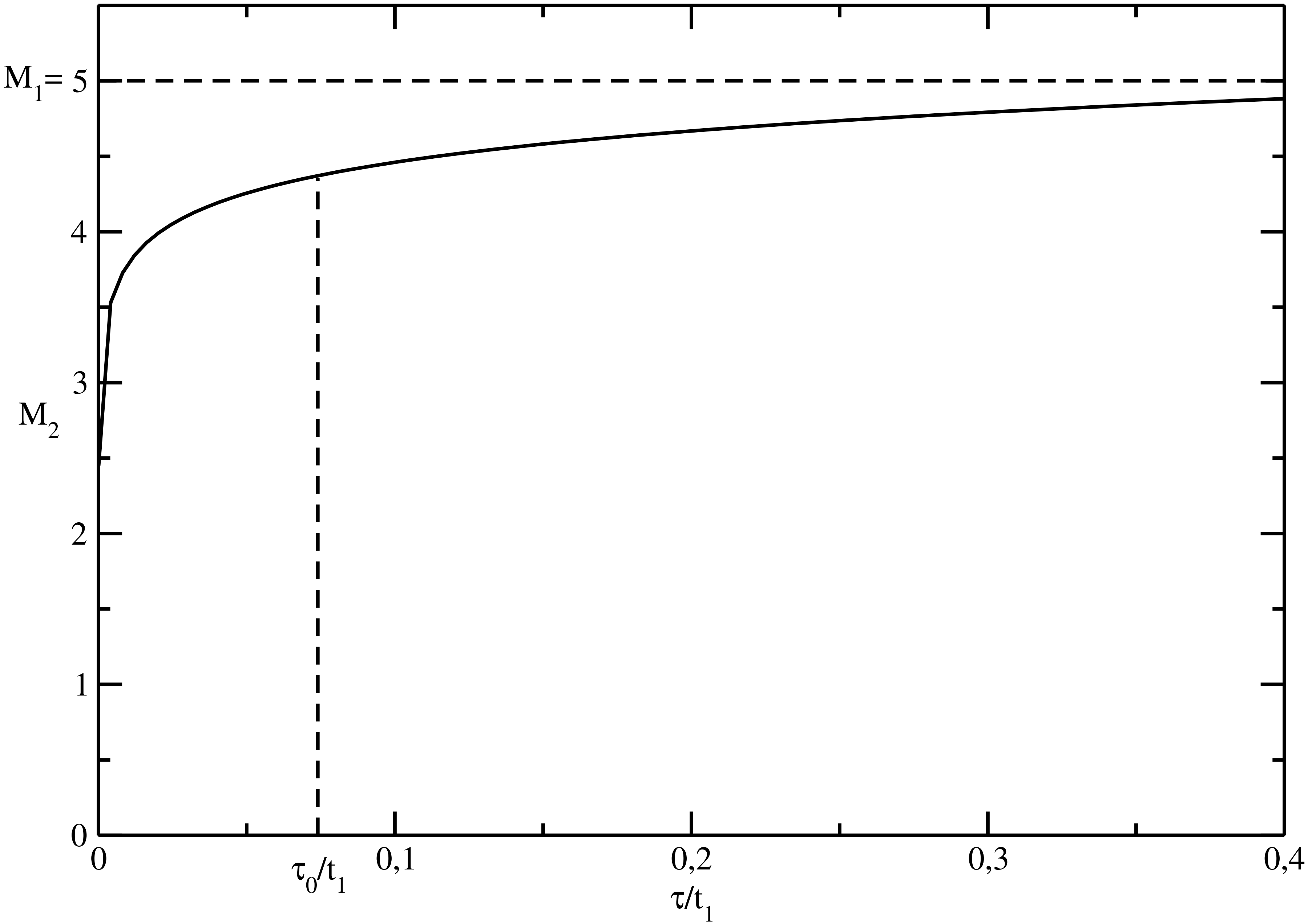
Utsu, T. (1069). Aftershocks and earthquake statistics (I,II): Source parameters which characterize an aftershock sequence and their interrelations. *J. Fac. Sci. Hokkaido Univ.*, Ser. VII, 2, 129-195, 196-266

Utsu, T. (2002). Statistical features of seismicity. *International Geophysics*, 81, Part A, 719-732

Vere-Jones, D. (1969). A note on the statistical interpretation of Bath's law. *Bull. Seismol. Soc. Amer.*, 59, 1535-1541

Vere-Jones, D. (2008). A limit theorem with application to Bath's law in seismology. *Adv. Appl. Prob.*, 40, 882-896

FigA.



M1= 5
4
3
M2
2
1
0
0
τ0/t1
0,1
0,2
0,3
0,4
τ/t1