Self-adjustment Laws of Fluvial Processes

Bao-shan FAN[1], Wan-guang Sun[2], Yu-hang Liu[2], and Cheng-zhen Li[2]

[1]China Water Northeastern Investigation, Design and Research Corporation Limited, Changchun, China
[2]China Water Northeastern Investigation, Design and Research Corporation Limited, Changchun

November 24, 2022

Abstract

The appearance of alluvial channel networks is similar to that of trees and their roots what have self-similarity. In the process of a fluvial evolution, alluvial channels are inner-shaping, adjust to achieve a stable equilibrium state, and form some channel patterns that establish balance relationships between the parameters of the local turbulence field of the channel flow and the supplies of the water discharge, sediment transport rate and sediment composition in a river basin. Therefore, the characteristic dimension lΩ of the representing eddy of the turbulence field of the channel flow and the dynamic characteristic length ld of a catchment are supposed to respectively represent the local dynamic factor and the dynamic factor stream along. The geometric dimension of the cross section of a channel is deduced. And then, it is clarified that the self-similarity equation of cross section morphology has similarity relationship conjugated with the self-similarity equation of the widths of a channel network. One of the hydraulic geometry relationship equations reveals that the nominal dimension lΩ and the gravitational acceleration g have the same effects on shaping channel patterns in the fluvial processes. Finally, the governing equations for channels in regime are derived. It is considered that the regime situation can be regarded approximately to that of the bankfull water level of channels, and exponential relationship equations of channels in regime are given systematically. All the results of the study are consistent with the actual statistical data of the channels of Songhua River Basin.

Self-adjustment Laws of Fluvial Processes

Bao-shan Fan[1,2]，Wan-guang Sun[1,2]，Yu-hang Liu[1,2]，and Cheng-zhen Li[1,2]

[1] China Water Northeastern Investigation, Design and Research Corporation Limited, Changchun, China.

[2] Research Center on Cold Region Engineering Ministry of Water Resources, Changchun 130061, China.

Corresponding authors: Bao-shan Fan (fanbaoshan@126.com)，Wan-guang Sun (sunwanguang@aliyun.com)

Key words: channel networks; morphodynamics; riverbed evolution; river regime; self-similarity

Key Points:

1) The self-similarity relationships of channel widths along streams and cross-section shapes is a pair of conjugated similarities.

2) An artificial virtual governing eddy controls the whole process of the bursting, energy transfer and dissipation in the turbulence field.

3) The relationship between the channel width and water discharge is the only one that is not affected by sediment supply of a river basin.

Abstract

The appearance of alluvial channel networks is similar to that of trees and their roots what have self-similarity. In the process of a fluvial evolution, alluvial channels are inner-shaping, adjust to achieve a stable equilibrium state, and form some channel patterns that establish balance relationships between the parameters of the local turbulence field of the channel flow and the supplies of the water discharge, sediment transport rate and sediment composition in a river basin. Therefore, the characteristic dimension l_Ω of the representing eddy of the turbulence field of the channel flow and the dynamic characteristic length l_d of a catchment are supposed to respectively represent the local dynamic factor and the dynamic factor stream along. The geometric dimension of the cross section of a channel is deduced. And then, it is clarified that the self-similarity equation of cross section morphology has similarity relationship conjugated with the self-similarity equation of the widths of a channel network. One of the hydraulic geometry relationship equations reveals that the nominal dimension l_Ω and the gravitational acceleration g have the same effects on shaping channel patterns in the fluvial processes. Finally, the governing equations for channels in regime are derived. It is considered that the regime situation can be regarded approximately to that of the bankfull water level of channels, and exponential relationship equations of channels in regime are given systematically. All the results of the study are consistent with the actual statistical data of the channels of Songhua River Basin.

Plain Language Summary

Except particularly younger ones, botanical trees around us have many grade dimensions of branches. Let's compare one part of the branches with the whole tree, we will find that they are very similar. The morphological similarity between one part and the whole of a tree is called self-similarity. The appearance of streams merging into rivers on the Earth in a bird's-eye view is similar to that of a botanical tree. Therefore, a river is also self-similar what just like a botanical tree. The dynamic characteristics of every river on the Earth are different. In order to grasp the laws of river evolution and changing processes, People have done a lot of exploration, but so far no ideal results have been obtained. The self-similarity of river channels, here, is used into the study of river evolution as a constrain equation, and a dynamic method for describing the turbulence field of water flow is proposed. In the paradigm of Newtonian mechanics, the kinetic relationships of river parameters are successfully given. These relationships can help us to understand the behavior of fluvial rivers, such as the self-sculpting of river patterns and various local and stream-along dynamic equilibriums.

1 Introduction

A river is a product of the interaction between its water flow and channel bed, which is achieved by means of sediment scouring and depositing on the bed. Sediment materials are sometimes a component of the water flow or sometimes a component of the channel bed. The water flow acts on and shapes the channel bed, meanwhile, the bed constrains the flow and affects the flowing structure. The two of the water flow and the bed are interdependent, inter-played, and mutually organized, and are always in the process of changing and adjusting. Therefore, the patterns of channel networks are the outcomes of the self-organizing and evolution according to certain dynamic principles.

There are two aspects of self-adjustment of an alluvial channel: local adjustment and stream-along adjustment. The local adjustment of a channel refers to the relationship between the cross-section geometry dimension and its morphology of the channel and the supplies of the water discharge, sediment and sediment composition. The stream-along adjustment of a channel refers to the relationship between the channel morphology and the channel dynamic behaviors such as scouring or silting and their balancing, branching and confluence along the course.

The study on the basic principles of the self-adjustment of a fluvial process has a long history, and many hypotheses are proposed, of which some representatives are the uniform distribution of energy dissipation and minimum total work in the system (Leopold, 1964), minimum variance (Langbein, 1964), minimum channel bed activity (Dou, 1964), minimum unit stream power (Yang , 1971), critical initiative condition (Li, 1976; Parker, 1978), minimum function of channel system(Chang, 1979), maximum energy dissipation rate (Huang, 1981), maximum sediment transport rate (White, 1982), maximum resistance coefficient (Davis, 1983) and so on. All these hypotheses derived a series of semi-theoretical and semi-experienced formulas, which cannot be proof rigorously and theoretically and may lead to conclusions incompatible with some observations. Therefore, the study of the principles of self-adjustment of the fluvial process is still a difficult task in river dynamics.

Channel networks are parts of the fluvial landforms and have self-similarity (Mandelbrot, 1983). The geometric shape with self-similarity of a channel formed by self-adjustment is mainly determined by the factors such as water discharge, the rate of sediment supply and the debris size. For the case where the channel flow's intensity is greater than the sediment starting condition and less than the suspending condition, only the bed load exists. When the flow intensity is greater than the suspending condition of sediment, the rate of bed load and the rate of suspended load increase and decrease synchronously. So the bed load transport is accompanied all time with the fluvial process, especially in the upstream part of a river basin, the transport of sediment is mainly bed load. The overall patterns of a river are principal components of the pool - riffle - bar unit and modes of bar development. In a fluvial process, the bar evolution is the essential connection between the episodic nature of bed material transport and the production of river morphology (Church M, and R. I. Ferguson, 2015). The bed load exchanges with the bed materials continuously in the process of the river evolution and inherently deform channel patterns laterally, such as straight, meandered or braided channels. And the channel morphology is an inevitable outcome of following the principles of self-adjustment, which is mainly trimmed and shaped by the bed load.

Based on the above understanding, the study on the principle of the self-adjustment of river channels is implemented in two aspects that are local adjustment and stream along adjustment, namely. It is proved that the self-similarity relationship of channel widths along streams and the self-similarity relationship of cross-section shapes is a pair of conjugated similarities. and the regime relationships of cross section parameters are given systematically.

2 Bed load transport rate in an alluvial river

2.1 Effective shear stress of sediment transport

The stream power of an alluvial channel is usually consumed in three aspects, which overcomes the form resistance of the channel, the shear stress of transporting sediment and the shear stress acting on the stationary particles on the channel bed namely. The shear stress of the water flow acts on sediment particles is called the effective shear stress in river dynamics. Einstein [1956] proposed that the effective shear stress τ_b' of water flow obeys the logarithmic law of turbulence, which is

$$\frac{u}{u_*} = 2.5\ln\left(30.2\frac{\chi y}{k_s}\right) \tag{2-1}$$

where u is the velocity of water flow, u_* is the shear velocity, k_s is the roughness element dimension of river bed, χ is the parameter for transition smooth-rough (Einstein gives the calculation curve).

For the convenience of calculation, Fan [1992, 1995] gives the fitting equations of the calculation curve of χ as

$$\begin{cases} \dfrac{k_s}{\delta} \le 0.2, \quad \chi = 3.5\dfrac{k_s}{\delta}; \\ 0.2 \le \dfrac{k_s}{\delta} \le 0.431, \quad \chi = 0.755\ln\left(\dfrac{k_s}{\delta}\right) + 1.923; \\ 0.431 \le \dfrac{k_s}{\delta} \le 2.0, \quad \chi = 1.613\cos\left(0.767\ln\dfrac{k_s}{\delta}\right); \\ \dfrac{k_s}{\delta} \ge 2.0, \quad \chi = 1.5\left(\dfrac{k_s}{\delta}\right)^{-2} + 1.0 \end{cases} \tag{2-2}$$

where $\delta = 11.6\dfrac{\nu}{u_*}$ is the nominal thickness of the boundary layer, ν is the kinematic viscosity coefficient of water.

The roughness of the bed surface in Eq. (2-1) is twice the particle size *d* of the sediment on a river bed which is the median particle size of bed load material. Averaging Eq. (2-1) along the water depth, yields

$$\frac{U}{\sqrt{gHJ}} = 2.5\ln\frac{11\chi H}{2d} \tag{2-3}$$

where *U* is the average velocity, *H* is the average water depth, *J* is the slop of water flow, and g is the gravity acceleration.

According to Einstein's (1950) recommendation, the formula for calculating the effective hydraulic radius R_b' of the effective shear stress τ_b' can be obtained by replacing *H* with R_b', this becomes

$$\frac{U}{\sqrt{gR_b'J}} = 2.5\ln\frac{11\chi R_b'}{2d} \tag{2-4}$$

In order to express simply, the effective shear stress τ_b' is written in dimensionless form θ_b'. Consequently,

$$\theta_b' = \frac{\tau_b'}{(\gamma_s - \gamma)d} = \frac{u_*'^2}{\dfrac{\gamma_s - \gamma}{\gamma} gd} \tag{2-5}$$

where γ_s is the specific gravity of sediment particles, γ is the specific gravity of water, u_*' is effective shear velocity .

The relationships among effective shear stress τ_b', effective shear velocity u_*', and effective hydraulic radius R_b' are $u_*' = \sqrt{\tau_b' / \rho} = \sqrt{gR_b'J}$, in which ρ is the density of the liquid.

In hydraulic calculations, the average water depth *H* of a channel is commonly used, and is not appeared in Eq. (2-4), but the slop *J* of water flow which is not easy to determine appears in it, which brings inconvenience to calculate the effective shear stress. Fan [1992, 1995] introduced the comprehensive capacity parameter for the alluvial channel flow

$$F_b = \left(\frac{U^2}{\frac{\gamma_s - \gamma}{\gamma} gd} \right)^{1.5} \Bigg/ \sqrt{\frac{UR_b}{\nu}} \tag{2-6}$$

where R_b is the hydraulic radius that is about equal to the average depth H ($R_b = H$) for a wide and shallow channel, ν is the kinematic viscosity coefficient of water.

The relationship [Fan, 1995] between the effective dimensionless shear stress θ_b' of channel flow and the comprehensive capacity parameter F_b is

$$\theta_b' = f'\left(\frac{\omega d}{\nu}\right) \cdot F_b^n \tag{2-7}$$

where ω is the terminal settling velocity of a single sediment particle in still clear water, n is the exponent that is given as

$$n = \begin{cases} 0.7; & \dfrac{\omega d}{\nu} < 1 \\ 0.8; & \dfrac{\omega d}{\nu} \geq 1 \end{cases} \tag{2-8}$$

And the function $f'\left(\frac{\omega d}{\nu}\right)$ is given as following

$$f'\left(\frac{\omega d}{\nu}\right) = \begin{cases} 0.145\left(\dfrac{\omega d}{\nu} + 0.01\right)^{0.152}; & \dfrac{\omega d}{\nu} < 15 \\ 0.048\left(\dfrac{\omega d}{\nu} - 8.8\right)^{0.38}; & \dfrac{\omega d}{\nu} \geq 15 \end{cases} \tag{2-9}$$

The curve of Eq. (2-7) has two parts that belong to smooth wall and rough wall [Fan, 2017] of water flow respectively, which is disconsecutive at $\frac{\omega d}{\nu} = 15$.

When calculating the effective dimensionless shear stress θ_b' with Eq. (2-7), the settling velocity of the medium sediment particle should be calculated by the formula of Zhang (1961), which is

$$\omega = \sqrt{\left(13.95\frac{\nu}{d}\right)^2 + 1.09\frac{\gamma_s - \gamma}{\gamma} gd} - 13.95\frac{\nu}{d} \tag{2-10}$$

2.2. Bed load transport of channel flow

The amount of bed load transport per unit time is called the rate of bed load transport and represented by the symbol "G_B" in kg/s or T/s, and the rate of bed load transport per unit width of a channel is commonly expressed in the symbol "g_b" in kg/s/m or T/s/m .

Let's consider there are *N* sediment particles in quantity moving on a channel bed in a unit area with a unit width. The average velocity of these *N* sediment particles along the flowing direction is V_s, the bed load transport rate per unit width should be

$$g_b = \rho_s \frac{\pi}{6} d^3 N V_s \tag{2-11}$$

The rate of bed load transport per unit width in dimensionless form is written as

$$\Phi_b = \frac{g_b}{\rho_s d \cdot \sqrt{\dfrac{\gamma_s - \gamma}{\gamma} gd}} \tag{2-12}$$

Fan [1995] proposed a method to solve the number of sediment particles *N* and their average velocity V_s along the flowing direction. And the Eq. (2-11) is expressed as

$$\Phi_b = 2(\theta_b' - \theta_c')\left(\varphi_0 \sqrt{\theta_b'} - 0.9\right) \tag{2-13}$$

where $\theta_c' = \dfrac{\tau_c'}{(\gamma_s - \gamma)d} = 0.045$ is the dimensionless effective shear stress in the critical condition of sediment particles on the channel bed from moving to stop [Fan, 2017], and $\phi_0 = 2.5\ln(11\chi)$ is the wall law coefficient of water flow.

From Eq. (2-13), a simple relationship with the wall law coefficient φ_0 can be seen that the rate of bed load transport per unit width in dimensionless form Φ_b is a function of the non-dimensional effective shear stress θ_b' only.

3 The local geometric dimension and shape of a cross-section

3.1. The local geometric dimension of a cross-section

The sediment from an eroded catchment enters river channels in the form of bed load materials. The channels automatically adjust themselves to certain geometries and dimensions to fit the runoff conditions of the river basin to transfer the supplied sediment.

For the bed load transportation of a channel, the sediment particles are big enough that generally $\dfrac{\omega d}{\nu} \geq 1$ is tenable, so *n* = 0.8 in Eq. (2-8), namely

$$\theta_b' = f'\left(\frac{\omega d}{\nu}\right) \cdot F_b^{0.8} \tag{3-1}$$

When the effective dimensionless shear stress θ_b' is much larger than the dimensionless effective shear stress in the critical condition of particles from moving to stop, $\theta_b' \geq \theta_c'$ in Eq. (2-13), The non-dimensional bed load rate is simplified as

$$\Phi_b \approx 2\varphi_0 \theta_b'^{1.5} \tag{3-2}$$

The relationship between the bed load transport rate per unit width g_b and the rate of bed load transport G_B of a channel is

$$g_b = \frac{G_B}{B} \tag{3-3}$$

where B is channel width.

The relationship between water discharge Q and the factors of a cross-section is

$$Q = BHU \tag{3-4}$$

where H is the average depth and U the average velocity of the cross-section.

For an alluvial channel, the cross sections are generally wide and shallow which are $\frac{B}{H} > 10$, the hydraulic radius R_b is approximately equal to the average depth H of the channel. Substituting of Eq. (3-1) in Eq. (3-2), yields

$$\frac{\sqrt{g}BH^{1.8}}{Qd^{0.3}}\left(\frac{G_B}{\gamma_s Q}\right)^{1/2} = \left(\frac{2\varphi_0\gamma}{\gamma_s - \gamma}\right)^{1/2}\left[f'\left(\frac{\omega d}{\nu}\right)\right]^{3/4} \Bigg/ \left(\frac{\sqrt{\frac{\gamma_s - \gamma}{\gamma} gd} \cdot d}{\gamma}\right)^{0.3} \tag{3-5}$$

When the particles of bed load materials are not very fine sand, for $\frac{\omega d}{\nu} \geq 15$, we obtain

$$\left[f'\left(\frac{\omega d}{\nu}\right)\right]^{3/4} \Bigg/ \left(\frac{\sqrt{\frac{\gamma_s - \gamma}{\gamma} gd} \cdot d}{\gamma}\right)^{0.3} \approx 0.38$$

For the full rough bed, the Einstein's parameter for transition smooth-rough is $\chi = 1$, and the wall law coefficient is $\varphi_0 = 2.5\ln(11) = 6$, thus Eq. (3-5) can be approximately written as

$$\frac{\sqrt{g}BH^{1.8}}{Qd^{0.3}}\left(\frac{G_B}{\gamma_s Q}\right)^{1/2} = 1.0 \tag{3-6}$$

From Eq. (3-6), the geometric dimension of a channel can be obtained with response to the water discharge, sediment discharge and particle size as

$$BH^{1.8} = Q\left(\frac{\gamma_s Q}{G_B}\frac{d^{0.6}}{g}\right)^{1/2} \tag{3-7}$$

The left side of Eq. (3-7) is the cross-section dimension $BH^{1.8}$ of a channel, and the water discharge at left of the right side of the equation presents the scale of the channel. The physical meaning of Eq. (3-7) is that the cross-sectional dimension $BH^{1.8}$ of a channel is completely determined by the hydrodynamic conditions of the water discharge, the rate of sediment supply and the debris size, which is a relationship between the local parameter $BH^{1.8}$ of a channel and the parameters along the channel.

3.2. The cross-section shape of a channel

From the perspective of a river basin, the parameter $\frac{\gamma_s Q}{G_B}$ in Eq. (3-7) is the volume of runoff required to produce unit volume of sediment in the basin, $\frac{\gamma_s Q}{G_B} d^{0.6}$ indicates the anti-erosion ability that is expressed by a parameter of the characteristic length l_d of the basin,

$$l_d = \left(\frac{\gamma_s Q}{G_B} \right)^{5/3} d \tag{3-8}$$

The parameter l_d is a stream-along factor. The bigger the sediment particles in the basin is, the stronger the anti-erosion ability is, and the larger l_d is. The more the volume of runoff required for the production of unit volume sediment $\frac{\gamma_s Q}{G_B}$ in the basin is, the stronger the anti-erosion ability is, and the larger l_d is.

Let's suppose that the shape of each cross section of a channel stream along is similar to each other by the mechanism of the self-adjustment, the dimension of the self similarity channel network is correspondence to Eq. (3-7) for the total sediment supply rate G_B in Eq. (3-8), we have

$$BH^{1.8} = \frac{l_d^{\,0.3}}{\sqrt{g}} Q \tag{3-9}$$

Generally, the relationship between the channel width and water flow discharge is considered as

$$B = \alpha_1 Q^{\beta_1} \tag{3-10}$$

where, α_1 is a coefficient; β_1 is an exponent.

There are two parameters in Eq. (3-10). And a constraint for Eq. (3-10) can be set. Here, let α_1 be a parameter independent of channel width B and depth H.

Substituting Eq. (3-10) into Eq. (3-9), there is

$$H^{1.8} = \frac{l_d^{\,0.3}}{\alpha_1 \sqrt{g}} Q^{1-\beta_1} \tag{3-11}$$

Dividing Eq. (3-10) by Eq. (3-11), yields

$$\frac{B}{H^{1.8}} = \frac{\alpha_1^{\,2} \sqrt{g}}{l_d^{\,0.3}} Q^{2\beta_1 - 1} \tag{3-12}$$

Here, $\frac{B}{H^{1.8}}$ that reflects the local cross-section shape of a channel is a parameter not related to B and H, and α_1 is a parameter not related to B and H, too. Thus the parameter l_d that is the anti-erosion ability of the basin and not related to B and H.

4 Stream-along adjustment principles of channels

4.1. Erosion or deposition along a channel

The water discharge Q is constant along a channel for steady flow without tributaries or river branches. The wear of the sediment particles along the stream is negligible if the reach is not particularly long, and the sediment particle size d is regarded as a constant. In order to study the shape changing of cross-section along the channel, Eq. (3-7) is differentiated along the direction of the stream, becomes

$$\frac{\partial\left(BH^{1.8}\right)}{\partial B}\frac{dB}{dl}+\frac{\partial\left(BH^{1.8}\right)}{\partial H}\frac{dH}{dl}=Q\left(\frac{\gamma_s Q}{G_B}\frac{d^{0.6}}{g}\right)^{1/2}\left(-\frac{1}{2}\right)\frac{1}{G_B}\frac{dG_B}{dl} \tag{4-1}$$

where l is the length of the channel.

According to Eq. (3-7), $Q\left(\frac{\gamma_s Q}{G_B}\frac{d^{0.6}}{g}\right)^{1/2}$ in Eq. (4-1) can be replaced by $BH^{1.8}$, this becomes

$$\frac{dB}{B}=-1.8\frac{dH}{H}-\frac{1}{2}\frac{dG_B}{G_B} \tag{4-2}$$

This is the differential equation of self-adjustment along a channel for steady flow.

When a channel is in scoured circumstances, the channel is getting deeper as some of the sediment of the bed are being washed away, and the water depth *H* is increasing along the stream. The rate of sediment transport G_B of the channel reach is increased consequently $dG_B > 0$. Therefore, from Eq. (4-2), the channel is getting narrower and deeper along the course $dB < 0$. As the channel width is getting narrower which is yielded by scouring, the two sides of the channel bank are not synchronized in the process of channel narrowing, which is often a unilateral growth of one bank side. As an outcome of the increase in the water depth of the channel and the unilateral growth of one shore, the channel is getting bent, which further promotes the erosion of the opposite shore and eventually forms a meandering channel. The meandering of the channel increases the path length of the stream, which reduces the channel slope and reduces the capacity of sediment transport, and makes an equilibrium of sediment transportation. The final bending shape of the channel depends on the need to increase the length of the stream during the self-adjustment.

Usually, the total width of the sluice gates of a dam on a channel is smaller than the channel width upstream the dam, $dB < 0$, which causes the channel to be soured downstream the dam. As the sediment siltation occurs upstream the dam, the water flow tries to recover the rate of sediment transport automatically and makes the channel be scoured downstream the dam. ($dH > 0$ and $dG_B > 0$). The recovery of sediment transport rate is slow and the scouring length of downstream is long. After the construction of the Three Gorges Dam in China, the general scouring of the middle and lower reaches of the Yangtze River is a realistic example of the principles of self-adjustment along a stream.

Contrary to the above, for the deposition circumstances of a channel reach, the sedimentation on the channel bed leads to decrease of the water depth *H* of the channel ($dH < 0$) and reduce the rate of sediment transport G_B ($dG_B < 0$). Therefore, from Eq. (4-2), it can be seen that at this time, the channel width increases along the stream of the siltation reach of the channel $dB > 0$. This makes the channel become scattered and shallow, such as the wandering reach of the Yellow River in China.

4.2. Equilibrium reach of sediment transport

For the equilibrium reach of sediment transport, $dG_B = 0$, the differential Eq. (4-2) of self-adjustment of channel morphology along the stream for steady flow becomes

$$\frac{dB}{B} = -1.8\frac{dH}{H} \tag{4-3}$$

This presents that one of the width or water depth is unchanged in the equilibrium reach of sediment transport, the other is also unchanged, that is, if $dB = 0$ then $dH = 0$, vice versa. If the channel width is changed by engineering activities, the depth of the water will adjust according to Eq. (4-3). Similarly, if the depth is changed, the width will adjust accordingly.

For example, bridge piers occupy the width of a channel on the construction of a bridge, which is caused $dB < 0$ by human beings. The result of self-adjustment is to increase the water depth H, $dH > 0$, which leads to bridge crossing scour commonly. The same effect will be produced in the downstream of longitudinal dike or spur dike for channel regulation.

4.3. Conjugated feature of stream-along self-similarity of channel networks

It has been noticed for a long time that the channel network of a river basin is similar to the branches of botanical trees. In fact, the morphology of channel networks is similar to that of tree roots which are all confluence growth and have self-similarity. Leonardo da Vinci claims in his NOTEBOOKS, note No.394, that "All the branches of a tree at every stage of its height when put together are equal in thickness to the trunk (below them)." Based on the fractal relationship between area and diameter, Mandelbrot [1983] introduces diameter exponent, which is expressed as

$$d_t^{\Delta} = d_{t1}^{\Delta} + d_{t2}^{\Delta} \tag{4-4}$$

where d_t is the diameter of botanical trees, and the exponent is $\Delta=2$. It is indicated that the fibrous bodies constituting a botanical tree have a uniform distribution in the tree trunk. Mandelbrot considers that the bankfull widths of the tributaries of a channel network are also in accordance with Eq. (4-4). The fractal features of the Mississippi River in the United States are studied by B. B. Mandelbrot, and the exponent $\Delta=2$ is obtained for the widths of the channels of Mississippi River, which is the same as the diameter exponent of botanical trees.

For the main channel widths and the widths of all the tributaries of a channel network that has self-similarity, and according to the definition of fractal geometry on similarity dimension, the self-similarity relationship of the channel widths in the form of exponent can be written as

$$B^D = \sum_{i=1}^{n} B_i^{D} \tag{4-5}$$

where D is the fractal dimension; $i = 1, 2, ..., n$; n is the number of tributaries.

Let's observe a simple channel network where two channels meet only, and marked by "1" and "2" for each. The water discharge of channel "1" is Q_1, and of channel "2" is Q_2. The water discharge of the main channel downstream the confluence of the channel "1" and "2" is Q, and according to the law of continuity, yields

$$Q = Q_1 + Q_2 \tag{4-6}$$

Changing Eq. (4-1) to

$$Q = BH^{1.8}\left(\frac{G_B}{\gamma_s Q}\frac{g}{d^{0.6}}\right)^{1/2} \tag{4-7}$$

Substituting Eq. (3-10) into Eq. (4-5), the simple channel network relationship before and after the confluence of channel "1" and "2" is

$$\left(\beta_1 Q^{\alpha_1}\right)^D = \left(\beta_{11} Q_1^{\alpha_1}\right)^D + \left(\beta_{12} Q_2^{\alpha_1}\right)^D \tag{4-8}$$

Comparing Eq. (4-6) with Eq. (4-8), since the two channels "1" and "2" are completely irrelevant, for the self-similarity simple channel network, therefore

$$\beta_1 = \beta_{11} = \beta_{12} \tag{4-9}$$

$$\alpha_1 = \frac{1}{D} \tag{4-10}$$

Put Eq. (3-10) into Eq. (4-9), there is

$$\frac{B^D}{Q} = \frac{B_1^{\ D}}{Q_1} = \frac{B_2^{\ D}}{Q_2} \tag{4-11}$$

Substituting Eq. (4-7) into Eq. (4-11), thus

$$\frac{B^{D\text{-}1} l_d^{\ 0.3}}{H^{1.8}} = \frac{B_1^{\ D\text{-}1} l_{d1}^{\ 0.3}}{H_1^{\ 1.8}} = \frac{B_2^{\ D\text{-}1} l_{d2}^{\ 0.3}}{H_2^{\ 1.8}} = l_\Omega^{\ D\text{-}2.5} \tag{4-12}$$

where $l_\Omega^{\ D-2.5}$ is a proportional parameter, which is written in an exponential form in order to ensure the dimensional harmony of Eq. (4-12), and l_Ω has a dimension of length.

Since channel "1" and "2" are completely arbitrary, it can be deduced from Eq. (4-12) that the parameter $l_\Omega^{\ D-2.5}$ is not related to the parameters $\frac{B^{D\text{-}1}}{H^{1.8}}$ and $l_d^{\ 0.3}$.Therefore, $\frac{B^{D\text{-}1}}{H^{1.8}}$ is not related to channel width *B* and water depth *H*. From Eq. (3-12), we know that $\frac{B}{H^{1.8}}$ has nothing with *B* and *H*, so there must be

$$D = 2 \tag{4-13}$$

As noticed, in order to deal a suitable transportation with the supplies of water and sediment upstream, the cross-section dimension of the channel is adjusted stream-along according to the principle of Eq. (3-7), so that the fractal dimension D of the width of the alluvial channel is equal to 2. Therefore, the self-similarity Eq. (4-5) of an alluvial channel network becomes

$$B^2 = \sum_{i=1}^{n} B_i^{\ 2} \tag{4-14}$$

The similarity relationship of the cross-sectional shape of the channel obtained from Eq. (4-12) is

$$\frac{B}{H^{1.8}} = \frac{1}{l_\Omega^{\ 1/2}}\frac{1}{l_d^{\ 0.3}} \tag{4-15}$$

Eq. (4-15) is also valid for the two unrelated channels "1" and "2", and will not be repeated hereafter.

Here, the parameter l_d needs to be further explained. Since Eq. (4-5) presents the self-similarity relationship of all channels in a river basin, which is a macroscopic overall concept, the parameter l_d in Eq. (4-15) should include the total water and sediment supplies of the basin, no matter whether the sediment transportation in channels is suspended load material and/or bed load material. That is, the sediment transport rate G_B in Eq. (3-8) is the sum of the transport rate of suspended load and bed load.

The two aspects of the self-similarity of a channel network have been revealed, one is the self similarity relationship of the channel width Eq. (4-14) stream-along, the other is the self similarity relationship of cross-section shape Eq. (4-15). They are similarity equations conjugated and synchronize with each other. One is true as another is demonstrated.

let's use the foot symbols of "1" and "2" again to indicate the two branches of a river in the same way as we do in Eq. (4-6). The water discharge of branch "1" is Q_1, and of branch "2" is Q_2, and the water discharge of the channel after the confluence of the two branches is Q. For a braided river with bifurcating channel streams, there are expressions of Eq. (4-5), (4-6) and (4-8), too. The upstream part of the branches is similar to tree branches, and the downstream part is similar to tree roots, all of which have self-similarity. Similarly, there are expressions of Eq. (4-14) and Eq. (4-15) for the branches in sediment transport equilibrium.

Both the aforementioned tributaries and branches have the self-similarity, which shows that the self-similarity of a channel network is the embodiment of the principles of the self adjustment along streams. The conjugated similarity equations of Eq. (4-14) and Eq. (4-15) express different meanings of channel similarity. Eq. (4-14) represents the self similarity of the channel pattern which belongs to the category of fractal; while Eq. (4-15) represents the dynamic similarity of the channel morphology which belongs to morphodynamic paradigm. As long as one of them is tenable, the other must be tenable, which shows the consistency of natural phenomenon and mechanism. The self similarity relationship of Eq. (4-14) is a phenomenon, while the dynamic similarity relationship of Eq. (4-15) is a mechanism which is one of the basic principles of the self adjustment of the alluvial morphology along streams.

According to Eq. (4-15), the morphological parameter $\frac{B}{H^{1.8}}$ of a channel is completely determined by the two parameters with length dimension: one is the local parameter l_Ω, which presents the influence of the turbulence field of water flow on the morphology of the cross section of the channel; the other parameter l_d is a stream-along factor that presents the influence of the water discharge, the rate of sediment supply and the debris size on the cross-section morphology of the channel.

Vortex clusters in the turbulence field of an open channel flow are transferring momentum and energy as the water flow in the channel from upstream to downstream. In the continuous cycle of turbulent activities of the channel flow, large momentum bodies with small-size eddies in the form of vortex clusters promote a series of bursting of big eddies from the channel bed during the processes of sweeping the bed surface, and make the transfer of momentum, then the big eddies later split themselves into smaller eddies step by step with energy consumption and momentum transfer. Very small- size eddies diffuse in the whole turbulence fluid finally. This cycling movement of the vortex clusters provides the required energy for the turbulence field, maintains the various functions of the channel flow, and ensures the local cross-section transportation of the water discharge and incoming sediment as well as the sediment composition. Therefore, the small- size eddies that directly affect the motion of sediment particles in and near the turbulence field should be clearly manifested in Eq. (4-15), in which the parameter l_Ω with length dimension should be directly proportional to the single representative eddy element dimension that dominates the turbulence field. So l_Ω is considered as the characteristic length of the dispersion representative eddies in the turbulence field.

Under the aforementioned recognition, for a steady uniform flow, the energy consumption per unit time and unit stream length is γQJ, which is transferred by eddies in turbulence field. So next, the turbulence field of channel flow is studied, which has interaction with the channel boundary that consists of sedimentation mainly.

5. Nominal eddies in turbulence field of a channel flow

5.1. The definition of "Ω" eddy

There is a dynamic relationship among the shape of a channel, turbulence field of the stream and the sediment transportation. The cross-section geometric dimension and morphology of the channel, which is the motion area of the vertexes in the turbulence field, are shaped by the water flow of the channel through sediment erosion and deposition. In the case of bankfull water level of the channel, the shape and geometric dimension of the turbulence field of the channel are the cross-section shape and geometric dimension of the channel. In order to study the interaction between the turbulence field and the boundary of a cross section, a way to describe the turbulence field in a dynamic manner has to be explored.

In the turbulence field, the larger vortex clusters are continuously converted to the smaller vortex clusters. Small-size eddies spread around, which diffuse to the whole flow field and is also a process of momentum transfer that provides the energy required for the various functions of the turbulence field. In order to quantitatively describe the dynamic characteristics of the turbulence field, it is assumed that the momentum transfer between the eddies in the turbulence field can be represented by the nominal eddies named "Ω" eddies that have characteristic length l_Ω. Each of the nominal eddies has a conical body that the diameter of its base area is equal to l_Ω, so the base area is equal to $\frac{\pi}{4}l_\Omega^2$, the height of the cone is equal to $\frac{1}{2}l_\Omega$, and the volume of the "Ω" eddy is equal to $\frac{\pi}{24}l_\Omega^3$.

For a steady uniform flow, the energy consumption per unit time and unit stream length is γQJ, which is achieved by the eddies transfer in the turbulence field. The sweeping water to the bed surface causes bursting of the water near the bed surface to generate a large number of eddies, turning the energy of the quantity γQJ into the kinetic energy of the eddies. The "Ω" eddies have the entire functions of the turbulence field, such as the energy consumption and momentum transfer. All the energy of the channel flow is transmitted through "Ω" eddies to form the spatial distribution of the energy in the turbulence field. So the average value of the kinetic energy of a single "Ω" eddy is $\frac{1}{2}\rho\frac{\pi}{24}l_\Omega^3U^2$ in the turbulence field, and the following equation can be established

$$\gamma QJ = M_\Omega \frac{1}{2}\rho\frac{\pi}{24}l_\Omega^3U^2 \qquad (5\text{-}1)$$

where M_Ω is the number of the "Ω" eddies that have the characteristic length l_Ω and are produced by the bursting water per unit time and unit stream length.

There are two unknown variables in Eq. (5-1), M_Ω and l_Ω. And a dynamic constraint can be set on one of them. Should the representative eddy (the "Ω" eddy) show the burst characteristics of the turbulence field, the characteristic length l_Ω of the "Ω" eddy is related to the average frequency p_Ω of intermittent germinations occurred in unit time in the turbulence field, which is expressed as

$$p_\Omega = \frac{1}{\sqrt{l_\Omega / g}} \tag{5-2}$$

After the "Ω" eddy in turbulence field has two dynamic constraints of Eq. (5-1) and Eq. (5-2), the definition of the "Ω" eddy in the turbulence field can be clearly given now: "Ω" eddy is an artificial virtual governing eddy in the turbulence field, which controls the whole process of the burst frequency, energy transfer and energy dissipation in the turbulence field.

5.2. The numbers and dimension of "Ω" eddies in turbulence field

For a steady uniform flow, the energy consumption per unit time and per unit stream length is γQJ, which is achieved by the vortex transfer in the turbulence field. The sweeping water to the bed surface causes bursting of the water near the bed surface to generate a large number of eddies, turning the energy of the quantity γQJ into the kinetic energy of the eddies.

For the water flow in the open channel, the parameter $\frac{\rho U u_*}{\tau_b} = \frac{\rho U}{\rho u_*}$ is used to indicate the strength level of the turbulence sweeping on the bed surface. The larger the parameter $\frac{\rho U}{\rho u_*}$ is, the stronger the sweeping on the surface is, and the more eddies are generated by one turbulence bursting from the surface. The Froude number $\frac{U^2}{g l_\Omega}$ indicates the activity degree of the eddies in the turbulence field. The larger the Froude number $\frac{U^2}{g l_\Omega}$ is, the more active the eddies are, and the quicker the eddies' attenuation is. Use the parameter $\frac{11\chi H}{2d}$ of Eq. (2-3) to represent the spatial scale of "Ω" eddies' activities along the water depth. Therefore, for the channel bed with a length of l and an area of Bl, the number M_Ω of the "Ω" eddies with a characteristic length l_Ω generated by the bursting in unit time is expressed as

$$M_\Omega l = \alpha^{D_\Omega + 1} \frac{\rho U}{\rho u_*} \frac{1}{\sqrt{l_\Omega / g}} \frac{g l_\Omega}{U^2} \left(\frac{\chi H}{2d} \right)^{D_\Omega} \frac{Bl}{\frac{\pi}{4} {l_\Omega}^2} \tag{5-3}$$

here, α is the spatial factor of the "Ω" eddy. D_Ω is the dimension of fractal space, which is the spatial dimension of the "Ω" eddy moving along the water depth in the turbulence field of a stream. $\frac{Bl}{\frac{\pi}{4}l_\Omega^{\ 2}}$ is the number of "Ω" eddies on the area of Bl.

Rearranging Eq. (5-3), the number of "Ω" eddies per unit time and unit stream length is

$$M_\Omega = \frac{4\alpha^{D_\Omega+1}}{\pi}\left(\frac{\chi H}{2d}\right)^{D_\Omega}\frac{Q}{Hu_*}\frac{1}{U^2}\left(\frac{g}{l_\Omega}\right)^{3/2} \tag{5-4}$$

Substituting Eq. (5-4) into (5-1), yields

$$l_\Omega^{\ 3/2} = \frac{12}{\alpha^{D_\Omega+1}}\left(\frac{2d}{\chi}\right)^{D_\Omega} H^{3/2-D_\Omega} J^{3/2} \tag{5-5}$$

In the turbulence field, the characteristic length l_Ω of a "Ω" eddy has nothing with water depth *H*, so the spatial dimension of the "Ω" eddy in the turbulence field is.

$$D_\Omega = \frac{3}{2} \tag{5-6}$$

Thus, the expression of the characteristic length l_Ω of the "Ω" eddy is

$$l_\Omega = \frac{12^{2/3}}{\alpha^{10/3}}\frac{2d}{\chi} J \tag{5-7}$$

The relationship between the channel slope and the characteristic length of the "Ω" eddy is

$$J = \frac{\alpha^{10/3}}{12^{2/3}}\chi\frac{l_\Omega}{2d} \tag{5-8}$$

It can be seen that the characteristic length l_Ω of a "Ω" eddy is directly proportional to the product of the average water depth and the channel slope. By defining equations (5-1) and (5-2), and recursive equations (5-4) and (5-7), the "Ω" eddies can be used to generalize the dynamic functions of the turbulence field of streams.

6. Morphodynamic relationships of an alluvial channel

6.1. Hydraulic geometry parameter of a cross section

The similarity relationship Eq. (4-15) of the cross-section shape of a channel includes two parameters, l_Ω and l_d, where l_Ω is a local parameter and l_d is a stream-along parameter. And the parameter l_Ω must exit in Eq. (3-12) of the local cross section shape. Because Eq. (3-12) and Eq. (4-15) should be the same, the undetermined parameter α_1 and exponent β_1 are, respectively

$$\alpha_1 = \left(\frac{1}{l_\Omega}\frac{1}{g}\right)^{1/4} \tag{6-1}$$

$$\beta_1 = \frac{1}{2} \tag{6-2}$$

From the derivation process of Eq. (3-12) and Eq. (4-15), it shows that the cross-section morphology $\frac{B}{H^{1.8}} = \frac{1}{l_\Omega^{0.5}} \frac{1}{l_d^{0.3}}$ of a channel integrates the local dynamic factor and the stream-along dynamic factor. Both Eq. (3-12) and Eq. (4-12) were derived based on the local dynamic relationship of the channel and the self-similarity stream-along. By comparing Eq. (4-12) with Eq. (3-12), the fractal dimension $D = 2$ of the channel width in Eq. (4-12) was obtained, and the undetermined parameters in Eq. (3-12) was determined by the feedback of Eq. (4-15). Eq. (3-12) and Eq. (4-12) confirm with each other and jointly determine the cross-sectional shape $\frac{B}{H^{1.8}} = \frac{1}{l_\Omega^{0.5}} \frac{1}{l_d^{0.3}}$ of the channel.

The empirical parameter $\frac{\sqrt{B}}{H}$ of regime channels is widely used in the engineering community which is very close to Eq. (4-15). It shows that the empirical parameter $\frac{\sqrt{B}}{H}$ is basically consistent with the similarity principles of channel network, and dynamic similarity Eq. (4-15) is a hydraulic geometry relationship of alluvial channels.

Since the dynamic similarity parameter $\frac{B}{H^{1.8}}$ is an embodiment of channel morphology, a morphological expression $\eta = \frac{B^{5/9}}{H}$ is defined here, which is more in line with the current habit of river engineering. According to Eq. (4-15), the definition is

$$\eta = \frac{B^{5/9}}{H} = \left(\frac{1}{l_\Omega^{1/2}} \frac{1}{l_d^{0.3}} \right)^{5/9} \tag{6-3}$$

For a full rough bed, the Einstein's parameter for transition smooth-rough is $\chi = 1$ in Eq. (3-7), the local representative roughness of sediment is the median particle d_b of the sediment on the channel bed surface. Thus,

$$l_\Omega = \frac{12^{2/3}}{\alpha^{10/3}} 2 d_b J \tag{6-4}$$

Substituting Eq. (6-4) into Eq. (6-3), yielding

$$\eta = \frac{B^{5/9}}{H} = \left(\frac{1}{l_d} \right)^{1/6} \left(\frac{\alpha^{10/3}}{12^{2/3}} \frac{1}{2 d_b J} \right)^{5/18} \tag{6-5}$$

That is to say, the hydraulic geometry parameter η of channel morphology is a function of the slope and the debris size of the local channel, whose dimension is the reciprocal of length power 4/9.

According to the data of Songhua River Basin in Table 1, the spatial coefficient of "Ω" eddy $\alpha = \frac{1}{6}$ is obtained; that is, Eq. (6-5) becomes

$$\eta = \frac{B^{5/9}}{H} = 0.1\left(\frac{1}{l_d}\right)^{1/6}\left(\frac{1}{d_b J}\right)^{5/18} \tag{6-6}$$

Table 1

Bankfull values of the main and tributaries of the Songhua River Basin

River	Station	Discharge (m^3/s)	Area (m^2)	Width (m)	Depth (m)	Debris size (mm)	Slope $\times 10^{-4}$	Note
Nenjiang	Jiangqiao	2049	1194	224	5.32	1.25	0.2	Main
Nenjiang	Dalai	1624	935	169	5.53	2.5	0.2	Main
Songhua River	Xiadaiji	2998	1699	272	6.24	1.1	0.3	Main
Songhua River	Haerb	3614	2215	427	5.18	0.25	0.5	Main
Songhua No.2	Fuyu	601	436	175	2.49	0.4	0.2	Tributary
Lalinhe	Caijiagou	516	361	148	2.44	0.7	0.2	Tributary
Yinmahe	Simajia	176	111	32	3.45	0.8	15.0	Tributary

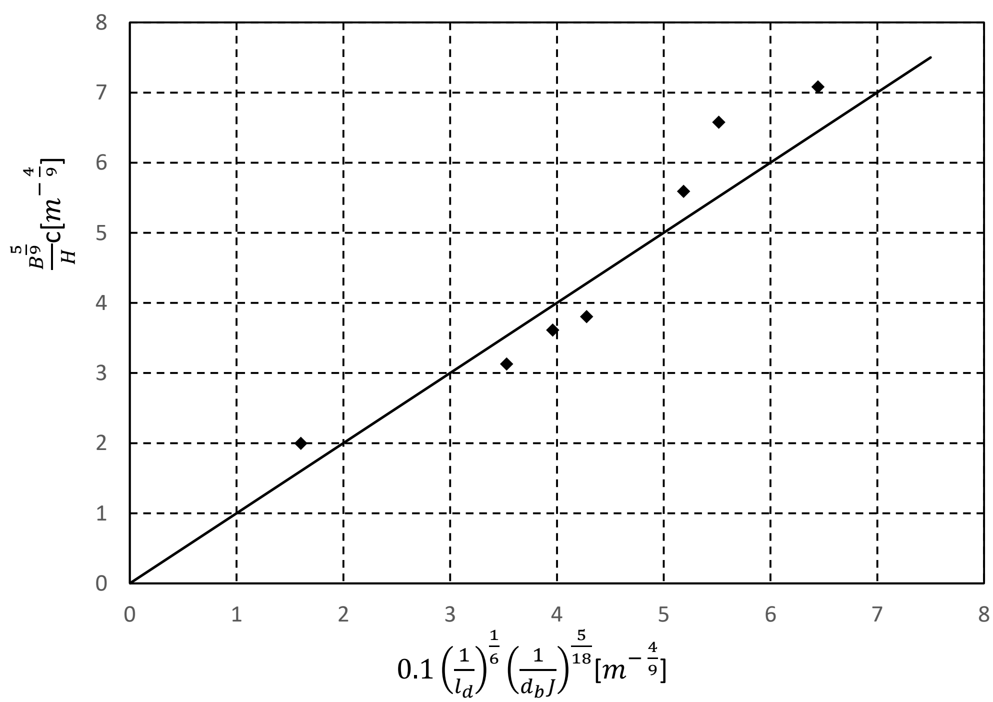

Figure 1. The hydraulic geometry parameter η in Songhua River Basin.

The correspondence between Eq. (6-6) and the data in Table 1 is shown in Figure 1. According to the sediment observation statistics of the hydrological stations in the Songhua River Basin, at the bankfull water level, the average particle diameter of the mainstream sediment transport in the Songhua River is about 0.048 mm, and the average length of the characteristic length l_d of the catchments is 700 m. The average particle diameter of sediment transported by Songhua No.2， Lalinhe and Yinmahe River is about 0.05mm, and the average length of the characteristic length l_d of the tow catchments is 400m.

6.2. The relationship between channel width and water discharge

Substituting Eq. (6-1) and Eq. (6-2) into Eq. (3-10), yielding

$$B = \left(\frac{1}{l_\Omega} \frac{1}{g} \right)^{1/4} Q^{1/2} \tag{6-7}$$

Eq. (6-7) shows that the characteristic length l_Ω of the " Ω " eddy in a channel flow stands by the gravity acceleration of the Earth, that is $\left(\frac{1}{l_\Omega} \frac{1}{g} \right)^{1/4}$, which states, in a fluvial processes, that the characteristic length l_Ω and the gravitational acceleration g has the same effects on shaping channel patterns. The relationship between the channel width and water discharge is the correlation between large-scale things, while the characteristic length of " Ω " eddy is a small-scale thing. And the large things' relationship is controlled by such a small-scale thing of " Ω " eddy, which makes one admire the charm of nature.

Substituting Eq. (6-4) into Eq. (6-7) gives

$$B = \left(\frac{\alpha^{10/3}}{12^{2/3}} \frac{1}{2 d_b J} \right)^{1/4} \left(\frac{Q}{\sqrt{g}} \right)^{1/2} \tag{6-8}$$

Eq. (6-8) reveals that the channel width has a fixed exponential relationship with the bankfull discharge, which is a power of 0.5, and is not influenced by the amount of sediment supply of the catchment.

Substituting the space coefficient $\alpha = \frac{1}{6}$ of the " Ω " eddy into Eq. (6-8), yields

$$B = 0.12 \left(\frac{1}{d_b J} \right)^{1/4} \left(\frac{Q}{\sqrt{g}} \right)^{1/2} \tag{6-9}$$

Eq. (6-9) is similar to the empirical formula $B = k\, Q^{0.5} / J^{0.2}$ given by C.T. Altunin who is well known in China in river engineering, and is tested with the statistical data in Table 1, see Figure 2, which fits well.

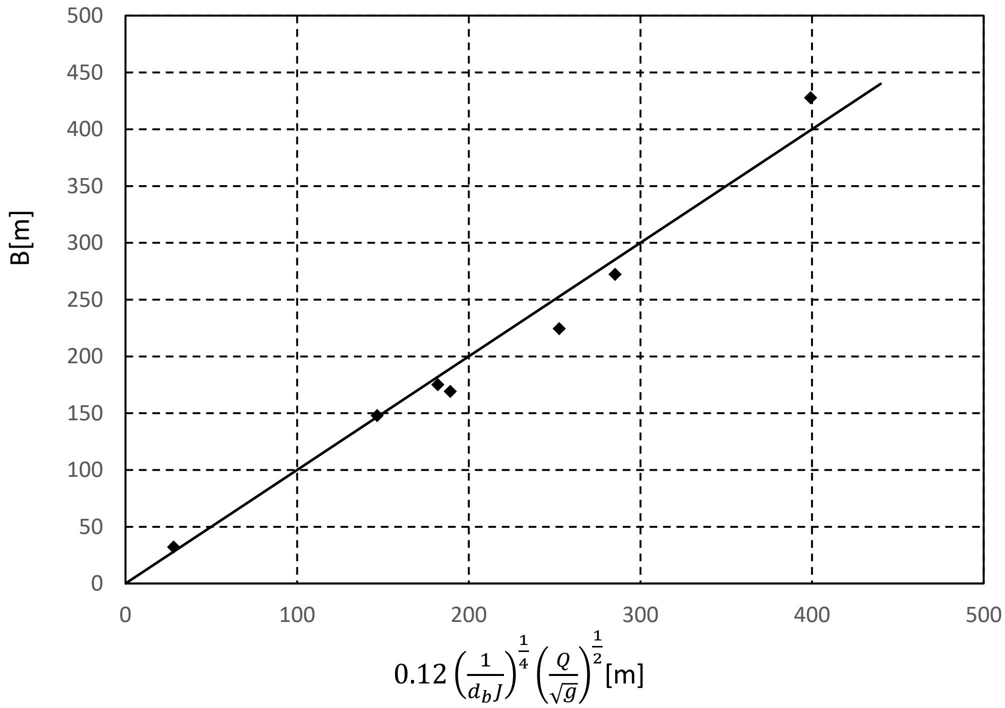

Figure 2. Channel width and bankfull discharge in Songhua River Basin.

6.3. The relationship between average flow velocity and water depth

According to the continuity formula $Q = AU$, Eq. (3-7) can be changed into

$$U = \left(\frac{H}{l_d}\right)^{0.3} \sqrt{gH} \qquad (6\text{-}10)$$

For a steady uniform flow and an equilibrium channel of sediment transportation, Eq. (6-10) gives a relationship between the average velocity and the average water depth of the cross section of an alluvial channel.

The famous Manning formula is:

$$U = \frac{1}{n} R^{2/3} J^{1/2} \qquad (6\text{-}11)$$

where n is the Manning roughness coefficient; R is the hydraulic radius in meters, m; U is the average velocity if channel flow in meters per second, m/s.

Substituting Eq. (6-11) and Eq. (3-8) into Eq. (6-11) with R=H, yields

$$\frac{R^{1/6} J^{1/2}}{n g^{1/2}} = \left(\frac{G_B}{\gamma_s Q}\right)^{1/2} \left(\frac{R}{d}\right)^{3/10} \qquad (6\text{-}12)$$

Equations (6-12) gives the relationship between the channel bed slope and the sediment supply per unit water discharge.

The tree morphodynamic relationships of Equations (6-3), (6-7), and (6-10) have their clear characteristics. Eq. (6-7) contains only the local dynamic factor l_Ω of the channel, Eq. (6-

10) only the dynamic factor l_d stream-along , and the hydraulic geometry parameter η of a cross section Eq. (6-3) is determined by the two factors l_Ω and l_d jointly.

7. Regime relationships of alluvial channel

7.1. Regime of alluvial channels

Due to the diversity of geological and geomorphological conditions and the hydrometeorological environment, channel networks in large river basins have formed various patterns. Only in vary small catchments with a single river, the river may have a single stable form. Regardless of whether it is a large or a small river basin, rainfall and soil erosion in the catchments provide the river(s) with incoming water and sediment and the sediment composition. In order to match such conditions of the incoming water and sediment and sediment composition, the channels in the basins evolve into corresponding channel morphology, so as to transport the corresponding water and sediment.

As erosion occurs in the river basin, sediment enters the channels. A process of sediment transportation by a channel is the process that the channel adjusts its self to achieve a balance between the local sediment transport capacity and the amount of sediment supply upstream. Therefore, the channel performance of regime is that the sediment transport capacity of the main channel stream is equal to the total sediment volume of its branches:

$$G_B = \sum_{i=1}^{n} G_{Bi} \tag{7-1}$$

in which *i = 1, 2, ..., n*; *n* is the number of tributaries.

For the simple channel network that only has two branch channels marked by "1" and "2", the sediment transport rate of channel "1" is G_{B1}, and of channel "2" is G_{B2}. The sediment transport rate of the main channel downstream the confluence of the channel "1" and "2" is G_B. In the case of the simple channel network in regime, we get

$$G_B = G_{B1} + G_{B2} \tag{7-2}$$

Changing Eq. (3-8) into

$$G_B = \gamma_s Q \left(\frac{d}{l_d} \right)^{3/5} \tag{7-3}$$

As mentioned above, the parameter l_d contains the total sediment supply, both of the suspended load and bed load.

Substituting Eq. (7-3) into Eq. (7-2), yielding

$$Q \left(\frac{d}{l_d} \right)^{3/5} = Q_1 \left(\frac{d_1}{l_{d1}} \right)^{3/5} + Q_2 \left(\frac{d_2}{l_{d2}} \right)^{3/5} \tag{7-4}$$

Substituting Eq. (4-6) into Eq. (7-4), yields

$$Q_1 \left(\left(\frac{d}{l_d} \right)^{3/5} - \left(\frac{d_1}{l_{d1}} \right)^{3/5} \right) + Q_2 \left(\left(\frac{d}{l_d} \right)^{3/5} - \left(\frac{d_2}{l_{d2}} \right)^{3/5} \right) = 0 \tag{7-5}$$

Due to the arbitrariness of the two channels "1" and "2", there are

$$\begin{cases} \left(\dfrac{d}{l_d}\right)^{3/5} - \left(\dfrac{d_1}{l_{d1}}\right)^{3/5} = 0 \\ \left(\dfrac{d}{l_d}\right)^{3/5} - \left(\dfrac{d_2}{l_{d2}}\right)^{3/5} = 0 \end{cases} \tag{7-6}$$

It can be write in the following form

$$\frac{l_d}{d} = \frac{l_{d1}}{d_1} = \frac{l_{d2}}{d_2} \tag{7-7}$$

Substituting Eq. (3-8) into Eq. (7-7), the governing equation of channels in regime is

$$\frac{\gamma_s Q}{G_B} = \frac{\gamma_s Q_1}{G_{B1}} = \frac{\gamma_s Q_2}{G_{B2}} \tag{7-8}$$

However, the incoming of water discharge and sediment and sediment composition is various continuously in years, seasons or even in days, and presents periodicity. But most channels on the Earth have relatively stable patterns corresponding to the bankfull discharge.

From the perspective of the whole basin, the bankfull discharge of a channel is a controlling and representative constant discharge that has an effective effect on the cross-sectional dimension and shape of the channel. Even for a large basin composed of several channel networks, when the bankfull discharge occurs in the main and tributary channels, the parameter l_d is often not very different as the geological and geomorphological conditions of the catchments in a same river basin are usually close as well as the hydrometeorological environment. Besides, the impact of the parameter l_d in Eq. (3-9) is low, whose power is 0.3. Therefore, a channel in the hydrodynamic condition at bankfull discharge can be approximately regarded as the channel in regime.

According Eq. (7-8), when a channel in regime, the parameter l_d is approximately a constant when the medium sediment particle of each channel is close to each other. Known with Eq. (3-9) in this case, the cross-sectional dimension $BH^{1.8}$ is proportional to the first power of the bankfull discharge Q.

A typical example of the channel networks in regime is the channels in Songhua River Basin located in the black soil area of Northeast China, with an annual runoff of 76.2 billion cubic meters and a drainage area of 565,800 square kilometers, of which there are 16 tributaries with an area exceeding 10,000 square kilometers. The rainfall of each catchment in the basin is very close, so as the vegetation coverage. Under the protection of vegetation, the basin has strong anti-erosion ability, and the water flow is known as a stream with less sand in China. The bankfull discharge of the channels of Songhua River Basin is about equal to the value of P = 50% flood peak or annual peak discharge of flood. The statistical data in Table 1 is the bankfull values of the main and its tributaries of the Songhua River Basin, see Figure 3 which indicates that the bankfull situation agrees well with the state of regime. The cross-section dimension of channels $BH^{1.8}$ is directly proportional to water discharge Q in the Songhua River Basin.

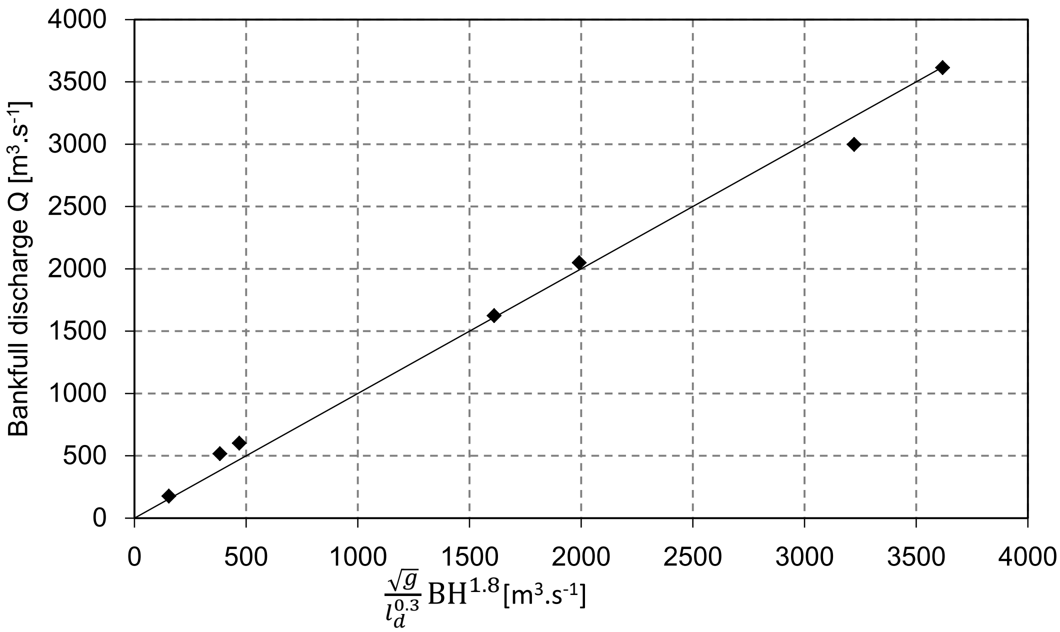

Figure 3. Bankfull relationship between Q and $BH^{1.8}$ of the Songhua River Basin

7.2. Width-depth ratio

The ratio of width to depth is a frequently used parameter in river engineering, which is obtained by Eq. (6-3) and Eq. (6-7)

$$\frac{B}{H}=\left(\frac{1}{l_{\Omega}}\right)^{7/18}\left(\frac{1}{l_d}\right)^{1/6}\left(\frac{Q}{\sqrt{g}}\right)^{2/9} \tag{7-9}$$

Substituting Eq. (6-4) into Eq. (7-9), obtains

$$\frac{B}{H}=0.04\left(\frac{1}{l_d}\right)^{1/6}\left(\frac{1}{d_b J}\right)^{7/18}\left(\frac{Q}{\sqrt{g}}\right)^{2/9} \tag{7-10}$$

Only the channels are in equilibrium, that is, the $\frac{G_B}{\gamma_s Q}$ of each channel is a constant, is the width-depth ratio $\frac{B}{H}$ of one of the channels directly proportional to the water discharge Q whose power is 2/9 in Eq. (7-10) because the dynamic factor l_d of a catchment is a function of $\frac{G_B}{\gamma_s Q}$.

7.3. Exponential relationships of a cross section

For the tree factors of the channel width, water depth and water flow velocity of a cross-section, Leopold, L. B., and Maddock, T., Jr. (1953) believe that the following simple exponential relationships exist between each of the tree factors and the water discharge of a stream as the river channel in regime.

$$\begin{cases} B = \alpha_1 Q^{\beta_1} \\ H = \alpha_2 Q^{\beta_2} \\ U = \alpha_3 Q^{\beta_3} \end{cases}$$

where, α_1, α_2 and α_3 are coefficients; β_1, β_2 and β_3 are exponents.

Eq. (6-8) is one of the simple exponential relationships which is the only exponential relationship not affected by sediment transport since β_1 is a constant and equal to 0.5, that is, the channel width and the water discharge have a fixed power of 0.5 whether the river is in regime or not. However, β_2 and β_3 are not constant in many cases, only when channels are in regime(the $\frac{G_B}{\gamma_s Q}$ are constants), be the tow exponents of β_2 and β_3 constants.

From equations (6-3) and (6-7), the relationship between water depth and flow discharge is

$$H = \left(l_\Omega^{1/2} l_d^{3/5} \frac{Q}{\sqrt{g}} \right)^{5/18} \tag{7-11}$$

Substituting Eq. (6-4) into Eq. (7-11) with $\alpha = \frac{1}{6}$, yielding

$$H = 3.18 l_d^{1/6} \left(d_b J \right)^{5/36} \left(\frac{Q}{\sqrt{g}} \right)^{5/18} \tag{7-12}$$

From the area calculation formula $A = BH$, the relationship between area and flow discharge can be obtained as

$$A = l_d^{1/6} \left(\frac{1}{l_\Omega} \right)^{1/9} \left(\frac{Q}{\sqrt{g}} \right)^{7/9} \tag{7-13}$$

Substituting Eq. (6-4) into Eq. (7-13) has

$$A = 0.4 l_d^{1/6} \left(\frac{1}{d_b J} \right)^{1/9} \left(\frac{Q}{\sqrt{g}} \right)^{7/9} \tag{7-14}$$

Substituting the formula $U = Q/A$ into Eq. (7-13), there is

$$U = \left(\frac{1}{l_d} \right)^{1/6} l_\Omega^{1/9} \sqrt{g} \left(\frac{Q}{\sqrt{g}} \right)^{2/9} \tag{7-15}$$

Substituting Eq. (6-4) into Eq. (7-15), gives

$$U = 2.52 \left(\frac{1}{l_d} \right)^{1/6} \left(d_b J \right)^{1/9} \sqrt{g} \left(\frac{Q}{\sqrt{g}} \right)^{2/9} \tag{7-16}$$

For the watersheds that have similar dynamic conditions that have the same erosion capacity for soil per unit runoff, the channels meet the condition of regime and the parameter $\frac{G_B}{\gamma_s Q}$ is constant. The average water depth H in Eq. (7-12), the cross section area A in Eq. (7-14) and the average velocity U of cross section in Eq. (7-16) have a fixed exponential relationship with the bankfull discharge Q. The exponential relationship between the average velocity and the bankfull discharge Q whose power is 2/9, so as the ratio of width to depth in Eq. (7-9). These equations are not independent, of which only one verified is enough. Eq. (7-14) is in good agreement with the data in Table 1, as shown in Figure 4.

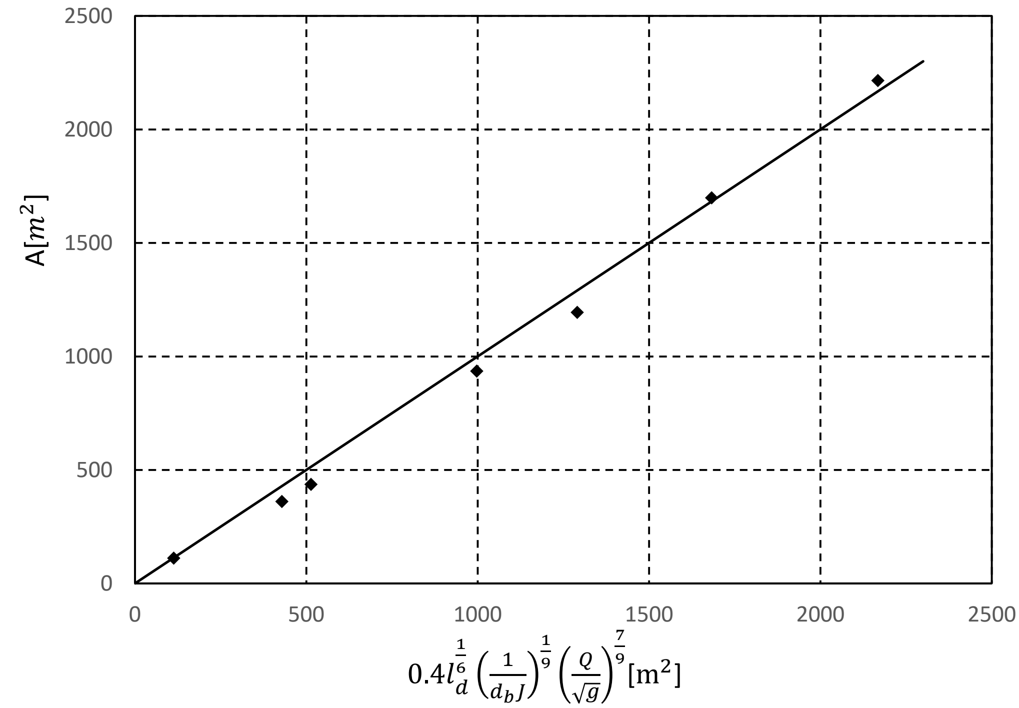

Figure 4. Cross-section area and bankfull discharge in Songhua River Basin.

8. Conclusion

The principles of self-adjustment of an alluvial channel are basic problems of river dynamics. Although the predecessors have explored in many aspects (Church, 2015), and introduced some assumptions that have some theoretical defects unfortunately. Based on the understanding that the self similarity phenomenon of channel networks is the result of the self adjustment of the alluvial channels to reach an equilibrium state, the principles of local and stream-along adjustment of channels are studied. The following laws of morphodynamics of fluvial channels are first clarified, which is of great significance to fluvial process.

1) Under the bankfull water level condition, channels are in regime in the catchments of similar dynamic erosion.

2) Eq. (6-8) on channel width and water discharge is the only one that is not influenced by sediment transport, which has an exponential relationship. The exponents in the relationships between other cross-section parameters and the water discharge are fixed only when $\frac{G_B}{\gamma_s Q}$ are constants for the regime.

3) The equation $\frac{B}{H^{1.8}} = \frac{1}{l_\Omega^{0.5}} \frac{1}{l_d^{0.3}}$ of the cross-section morphodynamic relationship of a channel integrates the tow impacts of the local dynamic factor and the stream-along factor.

4) The self-similarity of the channel width and the self-similarity of the cross-sectional shape are a pair of conjugated similarities, which are tow self adjustment principles of the channel morphology evolution of the alluvial river stream-along. The self-similarity $B^2 = \sum_{i=1}^{n} B_i^2$ of channel network is a representation, while the self-similarity $\frac{B}{H^{1.8}} = \frac{1}{l_\Omega^{1/2}} \frac{1}{l_d^{0.3}}$ of the cross-section shape is the mechanism that the turbulence field and gravity field play the same role in the process of channel morphology sculpturing.

Acknowledgment

Financial supported from the National Key Research and Development Program of China (Project No. 2018YFC0407303), and the provision of the lecture books about river basin erosion by Dr. Wen-hong Cao (China Institute of Water Resources and Hydropower Research) is gratefully acknowledged.

Datasets for this research are available in this in-text data citation reference: Bao-shan FAN et al., figshare (2020). doi.org/10.6084/m9.figshare.11782206.

References

Ackers P., W. R. White (1973). Sediment transport: new approach and analysis. *Journal of Hydraulic Division, American Society of Civil Engineers*, 99(11):2041- 2060.

Ackers, P. (1992), Gerald Lacey memorial lecture. Canal and river regime in theory and practice: 1929-92, *Ice Proceedings Water Maritime & Energy*, 96, 167– 178.

Bagnold R. A. (1977). Bedload transport by natural rivers. *Water Resources Research*, 13(2): 303 - 312.

Carbonneau, P., S. Lane, and N. E. Bergeron (2004). Catchment - scale mapping of surface grain size in gravel bed rivers using airborne digital imagery. *Water Resources Research*, 40. W07202. https://doi:10.1029/2003WR002759

Carling P.A., Cao Z., Holland M. J., Ervie D.A. and Babayan Koopaei K. (2002) Turbulent flow across a natural compound channel. *Water Resources Research*, AGU. 38, 6-1- 6-11.

Carson, M. A. (1984). The meandering - braided river threshold: A reappraisal. *Journal of Hydrology*, 73, 315–334.

Chang, H. H. (1979). Minimum Stream Power and River Channel Patterns. *Journal of Hydrology*, 41, 303–327.

Church, M., M. A. Hassan, and J. F. Wolcott (1998). Stabilizing self-organized structures in gravel-bed stream channels: Field and experimental observations. *Water Resources Research*, 34, 3169–3179.

Church, M. (2006). Bed material transport and the morphology of alluvial river channels. *Annual Review of Earth and Planetary Sciences*, 34, 325–354.

Church M, and R. I. Ferguson (2015). Morphodynamics: Rivers beyond steady state. *Water Resources Research*, 51(4), 1883–1897.

Coulthard, T. J. (2005). Effect of vegetation on braided stream pattern and dynamics. *Water Resources Research*, 41, W04003. https://doi:10.1029/2004/WR003201

Dade, W. B., and P. F. Friend (1998). Grain-size, sediment-transport regime, and channel slope in alluvial rivers. *Journal of Geology*, 106, 661–675.

Davis T. R. H., and Sutherland A. J. (1983). Extremal hypotheses for river behavior. *Water Resources Research*, 19(1), 141–148.

Dietrich, W. E., and J. D. Smith (1984). Bed load transport in a river meander. *Water Resources Research*, 20, 1355–1380.

Di Silvio, G., and M. Nones (2014). Morphodynamic reaction of a schematic river to sediment input changes: Analytical approaches. Geomorphology, 215, 74–82.

Dou, G. (1964). Channel bed evolution of alluvial rivers and tidal estuaries in plain. *Journal of Hydraulic Engineering*, (2), 1–13 (In Chinese).

Einstein, H. A. (1950). The bed-load function for sediment transportation in open channel flows, USDA Tech. Bull. 1026. Washington D.C.

Fan, B. (1992). A Theoretical Study on Sediment Transport, 5th International Symposium on River Sedimentation, KARLSRUHE, FRG, 4.

Fan, B. (1995). Kinematic study on fluvial sediment. *Journal of Sediment Research*, (3), 72–78 (In Chinese).

Fan, B., et al. (2017). Theory and application of sediment transport and ice load in open channel flow (In Chinese). *China Water & Power Press (CWPP)*, *Beijing*, 5.

Ferguson, R., and M. Church (2009). A critical perspective on 1 - D modeling of river processes: Gravel load and aggradation in lower Fraser River. *Water Resources Research*, 45, W11424. https://doi:10.1029/2009WR007740

Fiorentino M., et al. (1993). An entropy-based morphological analysis of river basin networks. *Water Resources Research*, 29(4), 1215–1224.

Gran, K., and C. Paola (2001). Riparian vegetation controls on braided stream dynamics. *Water Resources Research*, 37, 3275–3283.

Habersack, H. M., and J. B. Laronne (2001). Bed load texture in an alpine gravel bed river. *Water Resources Research*, 37, 3359–3370.

Harvey, A. M. (2002). Effective time scales of coupling within fluvial systems. *Geomorphology*, 44, 175–201.

Howard, A. D. (1967). Drainage analysis in geological interpretation, A summation. *American Association of Petroleum Geologists Bulletin*, 51, 2246–2259.

Huang W. (1981). The law of maximum rate the energy dissipation on continuum dynamics. *Journal of Tsinghua University*, (1): 87–96 (In Chinese).

Kleinhans, M. G., H. R. A. Jagers, E. Mosselman, and C. J. Sloff (2008). Bifurcation dynamics and avulsion duration in meandering rivers by one - dimensional and three - dimensional models. *Water Resources Research*, 44, W08454. https://doi:10.1029/2007WR005912

Langbein, W. B. (1964). Geometry of river channels. *Journal of the Hydraulic Division, American Society of Civil Engineers,* 90(HY2), 301–312.

Leopold, L.B., and Maddock, T., 1953. The hydraulic geometry of stream channels and some physiographic implications. *United States Geological Survey Professional Paper*, 252.Google Scholar

Leopold, L. B., M. G. Wolman and J. P. Miller (1964). Fluvial Processes in Geomorphology. W. M. Freeman and Co., 522 pages.

Li, R., D. D. Simons, and M. A. Stevens (1976). Morphology of cobble streams in small watersheds. *Journal of the Hydraulic Division, American Society of Civil Engineers,* 102(8), 1101–1118.
LI Y., J. DENG, and Z. SUN (2003). A study on river bed erosion downstream from the Tree Gorges Reservoir, *Journal of Basic Science and Engineering,* 11(3): 283-295.
Lom, A., J. S. Ribberink, and G. Parker (2008). Vertical sorting and the morphodynamics of bed form - dominated rivers: A sorting evolution model. *Journal of Geophysical Research*, 113, F01019. https://doi:10.1029/2006JF000618
Mandelbrot, B. B. (1983). The Fractal Geometry of Nature. San Francisco: Freeman, 156-163.
Millar, R. G. (2000). Influences of bank vegetation on alluvial channel patterns. *Water Resources Research*, 36, 1109–1118.
NI J., Y. LIU (2006), Ecological rehabilitation of damaged river system. *Journal of Hydraulic Engineering*, 37(9): 1029-1043 (in Chinese).
Ni J., L. Jin,et al (2002). Minimum water demand for ecosystem protection in the lower Yellow river. *Journal of Hydraulic Engineering*, （10）:1 – 7 (in Chinese).
Parker, G. (1978). Self-formed straight rivers with equilibrium banks and mobile bed: Part 2, The gravel rivers. *Fluid Mechanics*, 89(1), 127 – 146.
Shvidchenko A. B, G. Pender (2000). Flume study of the effect of relative depth on the incipient motion of coarse uniform sediments. *Water Resources Research,* 36(2): 619 - 628.
Slingerland, R., and N. D. Smith (2004). River avulsions and their deposits, A*nnual Review of Earth and Planetary Sciences*, 32, 257– 285.
Veitzer S.A., and V.K. Gupta (2001). Statistical self-similarity of width function maxima with implications to floods. *Advances in Water Resources*, 24(9-10): 955–965.
Wilcock, P. R., and J. C. Crowe (2003). Surface-based transport model for mixed-size sediment. *Journal of Hydraulic Engineering*, 129, 120 – 128.
White, W. R., Bettes R., and Paris E. (1982). Analytical approach to river regime. *Journal of the Hydraulic Division, American Society of Civil Engineers*, 108(HY10), 1179 –1193.
Yager, E. M., J. W. Kirchner, and W. E. Dietrich (2007). Calculating bed load transport in steep boulder bed channels. *Water Resources Research,* 43(7), W07418. https://doi:10.1029/2006WR005432
Yang, C.T. (1971). Potential Energy and Stream Morphology. *Water Resources Research*, 7(2), 312–322.
Zhang R., et al. (1961). The dynamics of river (In Chinese). *China Industry Press, Beijing*.