The effects of ice floe-floe interactions on pressure ridging in sea ice

Anders Damsgaard[1], Olga V. Sergienko[2], and Alistair Adcroft[3]

[1]Department of Geophysics, Stanford University
[2]AOS, Princeton University/GFDL
[3]Princeton University

November 24, 2022

Abstract

The mechanical interaction between ice floes in the polar sea-ice packs plays an important role in the state and predictibility of the ice cover. Using a Lagrangian-based numerical model we investigate the mechanics of sea ice floe-floe interactions. Our simulations show that elastic and reversible deformation offers significant resistance to compression before ice floes yield with brittle failure. When pressure ridges start to form, compressional strength dramatically decreases, implying thicker sea ice is not necessarily stronger compared to thinner ice. These effects are not accounted for in current sea-ice models that describe ice strength by thickness alone. As our results show, the observed transition in mechanical state during ridging initiation may lead to biases in simulated ridge building rates and sea-ice thickness. We propose a parameterization that describes failure mechanics from fracture toughness and Coulomb sliding, improving the representation of ridge building dynamics in particle-based and continuum sea-ice models.

Supporting Information for "Ice-floe mechanics and pressure ridging in sea ice"

A. Damsgaard[1], O. Sergienko[1], and A. Adcroft[1]

Contents of this file

1. Text S1 to S2
2. Figures S1 to S4
3. Table S1

Introduction

In this document we provide a full description of the two numerical models deployed in this study. Further results are included as supplementary figures. A summary table lists simulation parameters.

Corresponding author: A. Damsgaard, Department of Geophysics, 397 Panama Mall, Stanford University, Stanford, CA 94305, USA. (anders@adamsgaard.dk)

[1]Program in Atmospheric and Oceanic Sciences, Princeton University, Princeton, New Jersey, USA.

Text S1. Lagrangian modeling of compressive interaction between two ice floes

In order to investigate the mechanical interaction between ice floes, we design a Lagrangian numerical model based on the bonded discrete-element method [e.g., *Cundall and Strack*, 1979; *Potyondy and Cundall*, 2004; *Damsgaard et al.*, 2018; *Damsgaard*, 2018]. Non-bonded particles interact with a cohesion-less, elastic, and Coulomb-frictional rheology described in *Damsgaard et al.* [2018]. Bonded particles interact with elastic-plastic mechanics based on beam theory, after the 2D formulation by *Potyondy and Cundall* [2004]. The beam formulation resists relative rotation, shear, and tension between the particles. The tensile stress on a bond is limited by the ultimate tensile strength (σ_{uts}):

$$\sigma^{ij}_{\mathrm{uts}} > \frac{||\boldsymbol{f}^{ij}_{\mathrm{n}}||}{A^{ij}} + \frac{|M^{ij}_{\mathrm{t}}|R^{ij}}{I^{ij}}, \tag{1}$$

where M_{t} is the bending momentum on the bond:

$$M^{ij}_{\mathrm{t}} = \frac{k^{ij}_{\mathrm{n}} R^{ij}}{A^{ij}(r^i + r^j) I^{ij} \theta^{ij}_{\mathrm{t}}}. \tag{2}$$

I^{ij} is the particle-pair moment of inertia, approximated as,

$$I^{ij} = \frac{2}{3} R^3_{ij} \min(h_i, h_j), \tag{3}$$

and θ_{t} is the total relative rotation distance of the contact ($\theta^{ij}_{\mathrm{t}} = \int_t (\omega^j - \omega^i) dt$). The bond can also fail if shear stress exceeds the shear strength σ_{s}:

$$\sigma^{ij}_{\mathrm{s}} > \frac{||\boldsymbol{f}^{ij}_{\mathrm{t}}||}{A^{ij}} \tag{4}$$

If the bond stresses (right-hand sides of Eqs. 1 and 4) exceed the prescribed strengths (σ_{uts} and σ_{s}), the bond breaks and is no longer enforced. Bonds do not re-form in these simulations, except when simulating instant refreezing (Fig. S2).

Text S2. Plan-view ridging parameterization for larger-scale particle models

The ice-floe interactions transition from an elastic and reversible pre-failure state, to a ridged or rafting post-failure state when mechanical failure and vertical redistribution of the ice mass changes the physics of interaction (Fig. 2). In the following, we describe how the interaction of plan-view particles of ice-floe size is parameterized to include the ridging dynamics and rheological changes observed from the previous compressional experiments.

S2.1 Pre-failure contact mechanics

In the stage before compressive failure occurs, the contact rheology is parameterized from linear elasticity and Coulomb friction [*Damsgaard et al.*, 2018]. The contact-normal force $\boldsymbol{f}_\mathrm{n}$ is given by:

$$\boldsymbol{f}_\mathrm{n}^{ij} = -A^{ij} E^{ij} \boldsymbol{\delta}_\mathrm{n}^{ij} \tag{5}$$

The contact cross-sectional area between the cylindrical elements is defined as $A^{ij} = R^{ij} \min\left(h^i, h^j\right)$, where $R^{ij} = 2r^i r^j \left(r^i + r^j\right)^{-1}$ is the geometrical mean of the radii (Fig. 1a). E^{ij} is Young's modulus (the elastic modulus) for the contact. The contact-tangential (parallel) force $\boldsymbol{f}_\mathrm{t}$ is defined as,

$$\boldsymbol{f}_\mathrm{t}^{ij} = -\frac{E^{ij} A^{ij}}{R^{ij}} \frac{2(1-(\nu^{ij})^2)}{(2-\nu^{ij})(1+\nu^{ij})} \boldsymbol{\delta}_\mathrm{t}^{ij} \tag{6}$$

where $\boldsymbol{\delta}_\mathrm{t}^{ij}$ is the tangential displacement vector on the contact interface. This vector is incrementally calculated and corrected for contact rotation [*Damsgaard et al.*, 2018]. The magnitude of the contact-tangential force is limited by Coulomb friction:

$$||\boldsymbol{f}_\mathrm{t}^{ij}|| \leq \mu^{ij} ||\boldsymbol{f}_\mathrm{n}^{ij}|| \tag{7}$$

S2.2 Criteria for compressive failure

We find that the compressive failure limit in the detailed two-floe compressional experiments (Fig. 3) is well described by a relationship of the form,

$$||\boldsymbol{f}_\mathrm{n}^{ij} + \boldsymbol{f}_\mathrm{t}^{ij}|| \leq \min\left(K_\mathrm{Ic}^i, K_\mathrm{Ic}^j\right) \min\left(h^i, h^j\right)^{3/2} \quad (8)$$

where h is the ice-floe thickness and K_Ic is the fracture toughness (units Pa m$^{1/2}$, or N m$^{-3/2}$), characterizing the resistance to brittle failure. Note that the orthogonal normal ($\boldsymbol{f}_\mathrm{n}$) and tangential contact forces ($\boldsymbol{f}_\mathrm{t}$) can both contribute to the compressive stress on the contact. The 3/2-order dependency between thickness and strength is consistent with some previous parameterizations of ridging failure [e.g., *Rothrock*, 1975; *Hopkins*, 1998], but not the commonly used linear relationship [e.g., *Hibler*, 1979].

S2.3 Post-failure contact mechanics

After compressive failure has occurred (Eq. 8), the ice-floe contact is marked as actively ridging. The previous interaction mechanics (Eq. 5 to 7) are replaced with the parameterized ridging physics. After failure the ice floes are assumed to undergo stacking as a means of vertical rearrangement (Fig. 1b). Sliding friction along the sub-horizontal contact interface governs the mechanics in the post-failure state. The normal stress on the contact interface is determined by the hydrostatic response due to density differences and buoyancy:

$$\boldsymbol{\sigma}_\mathrm{n}^{ij} = (\rho_\mathrm{w} - \rho_\mathrm{i})(h_i + h_j)\boldsymbol{g}, \quad (9)$$

where ρ_w and ρ_i are the densities of water and ice, respectively, and $\boldsymbol{g}$ is the gravitational acceleration. The interficial tangential stress $\boldsymbol{\sigma}_\mathrm{t}$ is sub-horizontal, and is determined by the horizontal sliding distance $\boldsymbol{\delta}_\mathrm{s}$, the contact stiffness k_t, and the interface area A (Fig. 1b):

$$\boldsymbol{\sigma}_\mathrm{t}^{ij} = -k_\mathrm{t} \boldsymbol{\delta}_\mathrm{s}^{ij} A_{ij}^{-1} \tag{10}$$

We uphold the Coulomb-frictional limit on the sliding interface:

$$||\boldsymbol{\sigma}_\mathrm{t}^{ij}|| \leq \mu^{ij} ||\boldsymbol{\sigma}_\mathrm{n}^{ij}||, \tag{11}$$

and excess elastic energy is stored as frictional heat loss. Increases in contact strength by freezing can be added to the right-hand side of the above equation through a time and temperature-dependent cohesion term, but is not included here.

The normal and tangential forces on the particles are found by decomposing the tangential stress according to the contact orientation:

$$\boldsymbol{f}_\mathrm{n}^{ij} = (\boldsymbol{\sigma}_\mathrm{t}^{ij} \cdot \hat{\boldsymbol{n}}^{ij}) A^{ij} \tag{12}$$

$$\boldsymbol{f}_\mathrm{t}^{ij} = (\boldsymbol{\sigma}_\mathrm{t}^{ij} \cdot \hat{\boldsymbol{t}}^{ij}) A^{ij}, \tag{13}$$

where $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{t}}$ are unit-length normal and tangential vectors for the i and j particle pair. The forces grow non-linearly with increasing contact area during compression (Fig. 1b).

Table S1. Simulation parameters for ice-floe compression tests.

Parameter	Symbol	Value
Granular.jl software version	—	v0.3.4[a]
Ice particle radii	r	0.01 m
Young's modulus	E	2×10^7 Pa
Poisson's ratio	ν	0.285
Coulomb friction coefficient	μ	0.3
Maximum tensile strength	σ_c	400 kPa
Maximum bond shear strength	σ_s	200 kPa
Compressive velocity	c_v	$[0.05, 0.10, 0.2]$ m/s
Ice particle density	ρ_i	934 kg/m^3
Water density	ρ_w	1000 kg/m^3
Thickness in no. of grains for left ice floe	$n_{y,1}$	$[3, 5, 7, 9, 11, 13, 15, 17, 19, 21]$
Thickness in no. of grains for right ice floe	$n_{y,2}$	$[3, 5, 7, 9, 11, 13, 15, 17, 19, 21]$
Length in no. of grains for left ice floe	$n_{x,1}$	100
Length in no. of grains for right ice floe	$n_{x,2}$	100
Gravitational acceleration	g_z	-9.8 m/s^2
Numerical time step length	Δt	8.48×10^{-6} s
Simulation length	t_total	$\frac{n_{x,1}+n_{x,2}}{2} \frac{2r}{c_\text{v}}$

[a] See *Damsgaard* [2018] for DOI and downloadable snapshot.