# Effects of fluctuating magnetic field on the growth of the Kelvin-Helmholtz instability at the Earth's magnetopause

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### Abstract

At the Earth's magnetopause, the Kelvin-Helmholtz (KH) instability, driven by the persistent velocity shear between the magnetosheath and the magnetosphere, has been frequently observed during northward interplanetary magnetic field (IMF) periods and considered as one of the most important candidates for transporting and mixing plasmas across the magnetopause. However, how this process interacts with magnetic field fluctuations, which persistently exist near the magnetopause, has been less discussed. Here we perform a series of 2-D fully kinetic simulations of the KH instability at the magnetopause considering a power-law spectrum of initial fluctuations in the magnetic field. The simulations demonstrate that when the amplitude level of the initial fluctuations is sufficiently large, the KH instability evolves faster, leading to a more efficient plasma mixing within the vortex layer. In addition, when the spectral index of the initial fluctuations is sufficiently small, the modes whose wavelength is longer than the theoretical fastest growing mode grow dominantly. The fluctuating magnetic field also results in the formation of the well-matured turbulent spectrum with a -5/3 index within the vortex layer even in the early non-linear growth phase of the KH instability. The obtained spectral features in the simulations are in reasonable agreement with the features in KH waves events at the magnetopause observed by the Magntospheric Multiscale (MMS) mission and conjunctively by the Geotail and Cluster spacecraft. These results indicate that the magnetic field fluctuations may really contribute to enhancing the wave activities especially for longer wavelength modes and the associated mixing at the magnetopause.

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16	Key Points:						
17	• 2-D fully kinetic simulations of magnetopause Kelvin-Helmholtz instability initially						
18	imposing power-law field fluctuations are performed.						
19	• The growth of the instability especially for long wavelength modes is enhanced by the						
20	fluctuating field, leading to more efficient mixing.						
21	• Spectral features obtained from the simulations are in reasonable agreement with past						
22	spacecraft observations at the Earth's magnetopause.						
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### 24 Abstract

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At the Earth's magnetopause, the Kelvin-Helmholtz (KH) instability, driven by the 25 persistent velocity shear between the magnetosheath and the magnetosphere, has been frequently 26 27 observed during northward interplanetary magnetic field (IMF) periods and considered as one of the most important candidates for transporting and mixing plasmas across the magnetopause. 28 However, how this process interacts with magnetic field fluctuations, which persistently exist 29 near the magnetopause, has been less discussed. Here we perform a series of 2-D fully kinetic 30 31 simulations of the KH instability at the magnetopause considering a power-law spectrum of 32 initial fluctuations in the magnetic field. The simulations demonstrate that when the amplitude level of the initial fluctuations is sufficiently large, the KH instability evolves faster, leading to a 33 more efficient plasma mixing within the vortex layer. In addition, when the spectral index of the 34 initial fluctuations is sufficiently small, the modes whose wavelength is longer than the 35 theoretical fastest growing mode grow dominantly. The fluctuating magnetic field also results in 36 the formation of the well-matured turbulent spectrum with a -5/3 index within the vortex layer 37 38 even in the early non-linear growth phase of the KH instability. The obtained spectral features in the simulations are in reasonable agreement with the features in KH waves events at the 39 magnetopause observed by the Magntospheric Multiscale (MMS) mission and conjunctively by 40 the Geotail and Cluster spacecraft. These results indicate that the magnetic field fluctuations may 41 42 really contribute to enhancing the wave activities especially for longer wavelength modes and the associated mixing at the magnetopause. 43

### 45 **1. Introduction**

The Kelvin-Helmholtz (KH) instability has long been considered as an important process 46 for inducing efficient mass, momentum and energy transfer across sheared boundary layers in 47 collisionless plasmas. At the Earth's magnetopause, for example, the KH instability is driven by 48 the persistent velocity shear between the magnetosheath (shocked solar wind) and the 49 magnetosphere [e.g., Dungey 1955], and is believed to effectively contribute to forming the 50 Earth's low-latitude boundary layer (LLBL) where plasmas from magnetosheath and 51 52 magnetospheric origins are mixed [e.g., Nakamura et al., 2019 and references therein]. Indeed, 53 past in-situ spacecraft observations have revealed that surface waves and non-linear flow vortices, which could be generated by the KH instability, are frequently observed along the low-54 latitude magnetopause under steady and strong northward interplanetary magnetic field (IMF) 55 [e.g., Sckopke et al., 1981; Kokubun et al., 1994; Slinker et al., 2003; Kivelson & Chen, 1995; 56 57 Fairfield et al., 2000; Hasegawa et al., 2004, 2006; Foullon et al., 2008; Kavosi & Raeder, 2015; Moore et al., 2016]. 58

When the IMF is strongly northward, although the magnetic shear is too small across the 59 60 magnetopause and the shear layer is too thick to cause a rapid growth of spontaneous magnetic 61 reconnection, the flow of the KH vortex can locally compress the thick layer and induce fast reconnection [Nakamura et al., 2006, 2008]. A series of recent 2-D and 3-D fully kinetic 62 simulations have suggested that this so-called vortex-induced reconnection (VIR) process leads 63 to efficient plasma mixing within the vortex through the formation of multiple magnetic islands 64 65 (flux ropes) along the compressed layer [Nakamura et al., 2011, 2013; Daughton et al., 2014]. This vortex-induced flux rope formation has indeed been observed by the Time History of 66 Events and Macroscale Interaction during Substorms (THEMIS) spacecraft at the dusk-flank 67 magnetopause [Nakamura et al., 2013]. 68

More recently, based on full high-time-resolution fields and plasma data by the
Magnetospheric Multiscale (MMS) mission [Burch et al., 2016], electron and ion-scale VIR
signatures were directly observed for the first time at the postnoon magnetopause on 8
September 2015 [Eriksson et al., 2016a,b; Li et al., 2016; Vernisse et al., 2016]. Nakamura et al.
[2017a,b] successfully reproduced the observed VIR signatures in a 3-D fully kinetic simulation
of this MMS event. In the simulation, the VIR is first induced in the early non-linear growth

phase of the KH instability and the local VIR signatures in this phase, such as the structures of 75 the electron-scale central reconnection region called the electron diffusion region, the ion-scale 76 outflow jets and the Hall magnetic field, are quantitatively consistent with the observed 77 signatures [Nakamura et al., 2017, a,b; Sturner et al., 2018]. These consistencies indicate that the 78 observed KH waves at the MMS location (~16 magnetic local time, MLT) were in an early non-79 linear growth phase. Based on the consistencies, the simulation predicted that the VIR and the 80 subsequent turbulent formation of the flux ropes cause an efficient plasma mixing in later non-81 linear phases and globally forms a thick flank-to-tail LLBL during this event. 82

83 The simulation of the above MMS event further demonstrated that as the VIR develops, the spectral powers of the magnetic field and plasma flow fluctuations are significantly enhanced 84 across sub-ion and magnetohydrodynamic (MHD) scales [Nakamura et al., 2017a]. The spectral 85 index of the magnetic field fluctuations in the simulation in the early non-linear phase, in which 86 87 the simulation agree well with the MMS observations on the local VIR signatures, is close to -8/3 at MHD scales (i.e., larger than ion scales), while the more well-matured Kolomogorov-like 88 89 turbulent spectra with an index close to -5/3 appear in the later phases (see Figure 2 in Nakamura et al. [2017a]). However, using high-time-resolution MMS data, Stawarz et al. [2016] examined 90 turbulent fluctuations in this MMS event and showed well-matured turbulent spectra with an 91 index close to -5/3 at MHD scales. This indicates that there may be additional sources that drive 92 93 the turbulence even before the early non-linear growth phase of the KH instability during this MMS event. 94

95 In this paper, based on 2-D fully kinetic simulations of this MMS event, we examine effects of initially-fluctuating magnetic field on the growth of the KH instability and the 96 associated turbulence evolution within the vortex layer. Although the strong magnetic field 97 fluctuations have frequently been observed in the magnetosheath [e.g., Luhmann et al., 1986; 98 99 Alexandrova et al., 2008; Roberts et al., 2018], past simulation studies of the KH instability at the magnetopause have not considered these pre-existing fluctuations -i.e., setting up only 100 laminar conditions. Here we newly setup initial magnetic field fluctuations with turbulent spectra. 101 102 The new simulations demonstrate that the initial fluctuations enhance the growth of the KH instability over a broad range of scales, which leads to the formation of well-matured spectra 103 104 with a -5/3 index even in the early non-linear phase. The results reasonably agree with the observations of this MMS event as well as an earlier event in which Geotail and Cluster 105

simultaneously observed different amplitudes of the KH waves at different locations of the
 magnetopause [Hasegawa et al., 2009].

This paper is organized as follows. Section 2 describes the details of the simulation model employed in this paper. Section 3 presents the obtained simulation results, while in Section 4 we summarize the results and discuss the realistic role of the pre-existing magnetic field fluctuations on the KH instability through the comparison of the simulation results with past in-situ observations of the KH waves at the magnetopause.

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### 114 **2. Model**

# 115 2.1 Simulation settings

We perform a series of 2-D simulations of the KH instability induced at the 116 magnetopause, using the fully kinetic particle-in-cell code VPIC [Bowers et al., 2008, 2009]. The 117 initial simulation settings are the same as the ones employed in Nakamura et al. [2017a,b] except 118 for the in-plane magnetic field components, the tilt angle of the shearing flow from the direction 119 of the k-vector of the KH instability and the associated ion pressure balance across the velocity 120 shear boundary. In Nakamura et al. [2017a,b], the initial density, magnetic field and ion bulk 121 velocities across the magnetopause boundary layer were set to the values obtained from the 122 MMS observations before and after the KH wave interval 10:20-11:26 UT on 8 September 2015 123 124 [Eriksson et al., 2016]; denoting the high-density magnetosheath and low-density magnetospheric (boundary layer) sides as 1 and 2, respectively, the shearing flow  $U_1$ - $U_2$ =( $U_{x1}$ , 125  $U_{v1}, U_{z1} = (V_0 \cos\theta, 0, V_0 \sin\theta)$  along the magnetopause was slightly tilted  $\theta = 8.3^\circ$  from the x-126 direction, and the magnetic field varied from  $(B_{x1}, B_{y1}, B_{z1}) = (-0.1B_0, 0, B_0)$  to  $(B_{x1}, B_{y1}, B_{y1})$ 127  $B_{z1} = (0.2B_0, 0, B_0)$  across the boundary, where  $B_0$  is the strength of the uniform guide field. Here, 128 x-direction is along the k-vector of the fastest growing KH mode, y-direction is normal outward 129 to the magnetopause, and  $\mathbf{z}=\mathbf{x}\times\mathbf{y}$  completes the system. In this paper, to fit the 2-D study, the 130 flow tilt angle and the in-plane field variation are slightly modified to  $\theta=0^{\circ}$  and  $(B_{x1}, B_{x2})=(-1)^{\circ}$ 131  $0.15B_0$ ,  $0.15B_0$ ), respectively. The density ratio is set to be  $n_2/n_1=0.3$ , and the normalized 132 parameters are set to be  $n_1=n_0=25$  cm<sup>-3</sup>,  $B_0=74$  nT,  $|V_0|=355$  km/s=1.1V<sub>A</sub> based on  $n_0$  and  $B_0$ , as 133 employed in Nakamura et al. [2017a,b]. The initial density, field and flow components are set by 134

connecting the above values using a  $tanh(y/D_0)$  function [Nakamura & Daughton, 2014], where 135  $D_0=3.33d_i$  is the initial half thickness of the shear layer and  $d_i=c/\omega_{pi}$  is the ion inertial length 136 based on n<sub>0</sub>. To set the bulk velocities, particles are initialized with drifting Maxwellian 137 138 velocities. The electron temperature is set as uniform, while the ion temperature is set to satisfy 139 pressure balance, where the ion-to-electron temperature ratio in the magnetosheath is set to  $T_{i1}/T_{e0}$ =3.0. Additional electron and ion flows are set to satisfy the Harris type variation of  $B_x$ . 140 The electric field is set to satisfy  $\mathbf{E}$ =- $\mathbf{U}_{e} \times \mathbf{B}$  and the electron density near the current sheet is set 141 142 to be slightly higher than the ion density in the shear layer to satisfy the Gauss law [Pritchett and 143 Coroniti, 1984]. The ion-to-electron mass ratio is m<sub>i</sub>/m<sub>e</sub>=25 unless otherwise noted, and the ratio between the electron plasma frequency and the gyrofrequency based on  $n_0$  and  $B_0$  is  $\omega_{pe}/\Omega_e = 1.0$ . 144 The system size is set to be  $L_x \times L_y = 200d_i \times 300d_i = 4096 \times 6144$  cells with a total of  $2.5 \times 10^{10}$ 145 superparticles. The system is periodic in x, and y boundaries are modeled as perfect conductors 146 147 for the fields and reflecting for the particles.

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### 2.2 Initial perturbations

To investigate the effects of initially-fluctuating magnetic field, we add an ensemble of 150 magnetic field perturbations to the  $B_y$  and  $B_z$  components in the whole simulation domain as 151  $\delta B_0 = \sum_{1}^{1024} a(m_x) \cos(\frac{2\pi m_x}{L_x} - \varphi)$ , where m<sub>x</sub> is the mode number for the x-component of the k-152 vector (k<sub>x</sub>), and  $a(m_x)$  and  $\varphi(m_x)$  are the amplitude of the perturbation and the phase shift for 153 each m<sub>x</sub> mode, respectively.  $a(m_x)$  is set to be  $a(m_x)=a_0(m_x^{s}/4^{s})^{0.5}$  where  $a_0$  is the amplitude 154 for  $m_x=4$  (near the fastest growing mode of the KH instability [Miura & Pritchett, 1982]) and s is 155 the slope of the spectrum (i.e., the spectral index) of the initial fluctuations.  $\varphi(m_x)$  is set as 156  $\varphi(m_r) = 0.24191431 \times 2\pi m_r$  and  $\varphi(m_r) = 0.31142419 \times 2\pi m_r$  for B<sub>v</sub> ad B<sub>z</sub>, respectively, 157 for all modes to be out of phase. Note that this paper considers only the  $k_x$  component with 158 constant s between  $m_x=1$  and 1024. Effects of the other components as well as more realistic 159 spectral shapes will be investigated in future. 160

Table 1 shows the set of  $a_0$  and s we treated in this paper. Given that turbulent spectra with about -5/3 at MHD-scales were observed in the simulated MMS event in both the magnetosheath and boundary layer, as will be shown in section 4.2, we performed two runs in which *s*=-5/3 with different  $a_0$  ( $a_0$ =0.01B<sub>0</sub> for Run-B and  $a_0$ =0.001B<sub>0</sub> for Run-C). We also

performed four additional runs in which s=-1 (Run-D) and s=-1/3 (Run-E) with  $a_0=0.01B_0$  and 165 s=-11/3 (Run-F) and s=-8/3 (Run-G) with  $a_0=0.001B_0$ , and compared these results with the run 166 with  $\delta B_0 = 0$  (Run-A). The black curves in Figure 1 show 1-D power spectra of the initial  $B_v$ 167 fluctuations in the background region for run-B ( $\delta B_0^2 \propto k_x^{-5/3}$ ,  $a_0 = \delta B_0(m_x = 4) = 0.01B_0$ ) and run-C 168  $(\delta B_0^2 \propto k_x^{-5/3}, \delta B_0(m_x=4)=0.001B_0)$ . For both runs, as the simulations proceed, the amplitudes of 169 the modes for small-scales (sub-ion and smaller scales) are damped down to the PIC noise level, 170 while the amplitudes for larger-scales are not significantly changed. Although these background 171 spectra are more or less different from those in the MMS observations (see Figure 12) as well as 172 those in past magnetosheath observations [e.g., Alexandrova et al., 2008], this is the first 173 numerical challenge to investigate the growth of the KH instability in a non-laminar setup in the 174 175 fully kinetic regime.

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### 177 **3. Simulation results**

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3.1 Overview of the simulation results

Figures 2 and 3 show the time evolution of 2-D contours of the mixing fraction for ions 179  $F_i = (n_{i1} - n_{i2})/(n_{i1} + n_{i2})$ , which highlights the surface of plasma mixing between two regions [e.g., 180 Daughton et al., 2014; Nakamura & Daughton, 2014], for runs-A, B, C and E. For  $\delta B_0=0$  (Run-181 A), the evolution of the surface waves for  $m_x \sim 4-6$  modes, which correspond to the wavelength 182  $(\lambda \sim 4\pi D_0)$  of the theoretical fastest growing KH mode in the ideal MHD regime [Miura & 183 Pritchett, 1982], is visibly seen after t~ $100\Omega_i^{-1}$ . The formation of non-linear vortices of these 184 modes is seen at t~250-300 $\Omega_i^{-1}$  (Figure 2b), and these vortices merge into larger ones in the later 185 time (Figures 2c and 2d). Past MHD and multi-fluid studies suggested that the flow of the non-186 linear KH vortex locally compresses pre-existing magnetic shear layers and can induce magnetic 187 reconnection [Pu et al., 1990; Knoll & Chacon, 2002; Nakamura et al., 2006, 2008]. Past kinetic 188 simulations further demonstrated that this so-called vortex-induced reconnection (VIR) forms 189 multiple magnetic islands (flux ropes) along the edge of the vortices [Nakamura et al., 2011, 190 2013]. These islands are merged into the vortex bodies and cause rapid mixing within the 191 vortices as seen in Figures 2c and 2d and recently reported in our kinetic simulations [Nakamura 192 et al., 2011, 2014, 2017a, b]. 193

For run-B ( $\delta B_0^2 \propto k_x^{-5/3}$ ,  $\delta B_0(m_x=4)=0.01B$ ), the evolution of a broad range of modes 194 proceeds faster than those for  $\delta B_0 = 0$  (Figures 2e-2h). In particular, the longer wavelength modes 195 with  $m_x \sim 2$  grow more dominantly than the theoretical fastest growing mode  $m_x \sim 4-6$  modes and 196 are visibly seen even at t~100 $\Omega_i^{-1}$  (Figure 2e). The vortex structure in the early non-linear phase 197 is more complicated with sub-structures of a broad range of scales than that for  $\delta B_0=0$  (compare 198 199 Figures 2b and 2f). In the later phases, the long wavelength modes with  $m_x \sim 1-2$  grow dominantly (Figure 2g). The growths of these modes are stronger than those for  $\delta B_0=0$ , leading to formation 200 of a thicker mixing layer than for  $\delta B_0=0$  (compare Figures 2d and 2h). 201

For run-C ( $\delta B_0^2 \propto k_x^{-5/3}$ ,  $\delta B_0(m_x=4)=0.001B$ ), no clear fluctuation is seen at t=100 $\Omega_i^{-1}$ , and 202 the non-linear vortex structures for the theoretical fastest  $m_x \sim 4-6$  modes dominantly grow at 203 t=295 $\Omega_i^{-1}$  (Figures 3a and 3b), as also seen for  $\delta B_0=0$ . However, unlike in the case of  $\delta B_0=0$ , the 204 wavy structures for longer wavelength modes are visibly seen at t= $295\Omega_i^{-1}$  (compare Figures 2b 205 and 3b). In the later phases, these long wavelength modes with  $m_x \sim 1-2$  grow more strongly and 206 207 lead to formation of a thicker mixing layer than  $\delta B_0=0$  (compare Figures 2d and 3d). For run-E  $(\delta B_0^2 \propto k_x^{-1}, \delta B_0(m_x=4)=0.01B)$ , the overall vortex evolution is similar to Run-B, but smaller-208 scale modes grow faster than for Run-B (compare Figures 2e-2h and 3e-3h). This could be 209 210 because the amplitudes of the initial fluctuations for the smaller scale modes for Run-E are larger than those for Run-B. 211

Figure 4 shows time evolutions of the averaged thickness of the mixing layer in the boundary normal (y) direction. For  $\delta B_0=0$  (Run-A), the thickness starts increasing after the onset of the VIR in the early non-linear growth phase of the KH instability (t~250-300 $\Omega_i^{-1}$ ). For finite  $\delta B_0$  with large amplitudes (Runs-B and E), followed by the earlier onset of the KH instability, the mixing starts being enhanced earlier than that for the weak (Run-C) or no  $\delta B_0$  (Run-A). However, even for the weak  $\delta B_0$  (Run-C), the mixing progresses faster than that for  $\delta B_0=0$  in the later non-linear phases, since the long wavelength modes grow faster than for  $\delta B_0=0$ .

#### 3.2 Mode competition 220

Figure 5 shows time evolutions of 1-D spectra of U<sub>iv</sub> modes for runs-A, B, C and E 221 (corresponding to the runs shown in Figures 2 to 4). For  $\delta B_0=0$  (Figure 5a), the clear peak of the 222 theoretical fastest growing KH mode appears at  $m_x \sim 5$  at  $t \sim 150 \Omega_i^{-1}$ , and then the peak shifts to 223 longer wavelength modes as the vortex merging progresses as seen in Figures 2a-2d. The 224 amplitudes for smaller scale modes are enhanced after t~ $250\Omega_i^{-1}$  corresponding to the formation 225 of the small-scale island by the VIR. 226

For run-B (Figure 5b), corresponding to the initial fluctuations, larger amplitudes of the 227 U<sub>iv</sub> fluctuations are seen at longer wavelength modes in the initial phase. A clear peak of the long 228 wavelength modes with  $m_x=2$  becomes visible at t~100 $\Omega_i^{-1}$ . Then, the amplitudes for smaller 229 scale modes are enhanced after t~150 $\Omega_i^{-1}$ . After that, the longest wavelength mode (m<sub>x</sub>=1) is 230 visibly enhanced from t~ $200\Omega_i^{-1}$  On the other hand, for run-C (Figure 5c) in which the 231 amplitude level of the initial fluctuations is set to be one order of magnitude smaller than for run-232 B, the amplitudes of the U<sub>iv</sub> fluctuations are much weaker than for run-B in the initial phase, and 233 start being visibly enhanced at  $m_x=2-8$  after t~100 $\Omega_i^{-1}$ . The strong peak of the theoretical fastest 234 growing mode ( $m_x \sim 5$ ) appears at t~250-300 $\Omega_i^{-1}$ . Then, the peak shifts to the longer wavelength 235 modes and the amplitudes for smaller scale modes are enhanced as seen for run-A. However, the 236 time-scale of the inverse cascade (peak shifting) for run-C is somewhat shorter than that for run-237 A. The peak amplitude of the longer wavelength modes at  $m_x=1-2$  is also somewhat stronger 238 than that for run-A. 239

For run-E (Figure 5d) in which the amplitude level of the initial fluctuations  $(a_0=0.01B_0)$ 240 is the same as run-B but the spectral index (-1/3) is larger than run-B, the spectrum in the initial 241 phases spreads more widely than run-B. The clear peak near the theoretical fastest growing mode 242  $(m_x \sim 4-5)$  appears at t~100-150 $\Omega_i^{-1}$ . Then, the peak shifts to the longer wavelength modes and the 243 amplitudes for smaller scale modes are enhanced as seen for run-A. However, the time-scale of 244 the inverse cascade for run-E is much (almost two times) shorter than that for run-A. The peak 245 amplitude of the longer wavelength modes at  $m_x=1-2$  is also somewhat stronger than that for run-246 247 A.

Figure 6, which shows time evolution of the selected  $U_{iv}$  modes at  $m_x=1,2,5$  and 10, 248 highlights the differences in the growth of the MHD scale modes seen in Figure 5. For run-A 249 (Figure 6a), a typical inverse cascade process of the KH instability, which corresponds to the 250 vortex merging process as seen in Figures 2a-d, starts from the theoretical fastest growing  $m_x=5$ 251 mode. For run-B (Figure 6b), the longer wavelength  $m_x=1$  and 2 modes are initially enhanced 252 and grow more strongly than the theoretical fastest growing  $m_x=5$  mode. After the initial 253 enhancement of the two modes, the  $m_x=2$  mode dominantly grows, and after t~600 $\Omega_i^{-1}$  the  $m_x=1$ 254 255 mode becomes dominant. For run-C (Figure 6c), although the  $m_x=5$  mode grows first as is the case for run-A, the  $m_x=2$  mode starts growing earlier than run-A. As a result, the  $m_x=2$  mode 256 becomes dominant about 50  $\Omega_i^{-1}$  earlier than run-A. For run-E (Figure 6d), although the 257 evolution curve of the m<sub>x</sub>=2 mode is similar to run-B, the m<sub>x</sub>=5 mode dominates until t~200  $\Omega_i^{-1}$ . 258 As a result, the inverse cascade from the  $m_x=5$  to 1 modes occurs as in the case of Run-A, but it 259 proceeds about two times faster than run-A. These differences result from the different amplitude 260 of the initial fluctuations for each mode and the resulting mode competitions as below. 261

The evolution of each mode for  $\delta B_0=0$  is spontaneously induced by the background 262 random PIC noise. We confirmed that the growth rate of each mode for  $\delta B_0=0$ , measured from 263 the slope of the time evolution of  $U_{iv}^{2}$  for each mode in the linear growth phase, is consistent 264 with the rate in the ideal MHD regime [Miura & Pritchett, 1982] (not shown). For the finite  $\delta B_0$ , 265 although the growth rate of each mode is not significantly different from the one for  $\delta B_0=0$  (for 266 example, compare the slopes of the blue curves in Figure 6), the initial magnetic field 267 fluctuations additionally enhance the initial amplitude of the corresponding U<sub>iv</sub> modes. A similar 268 enhancement of the normal component of the flow velocity in the initial phase is also seen in 269 recent MHD simulations of the KH instability considering flow perturbations for specific modes 270 [Nykyri et al., 2017]. Assuming that the linear growth rate of each  $U_{iv}$  mode  $\gamma(m_x)$  is the same as 271 the one for  $\delta B_0=0$ , the time-scale  $\Delta t(m_x)$  during which the initially enhanced amplitude of a 272 273 mode  $b(m_x)$  grows and reaches a specific level  $c (\sim b \cdot \exp(\gamma \Delta t))$  is described as

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$$\Delta t(m_{\chi}) \sim \frac{\ln[c/b(m_{\chi})]}{\gamma(m_{\chi})}. (1)$$

Here, we discuss the mode competition between the theoretical fastest growing  $m_x=5$ mode and a longer wavelength  $m_x=2$  mode. Assuming that the initially enhanced amplitude  $b(m_x)$  is proportional to the amplitude of the initial field fluctuation for each mode, defining

278  $b_0=b(5), b(2)=b_0(5/2)^{|s|}$ , where *s* is the spectral index of the initial fluctuations. Since the growth

rate for  $m_x=5$  is about two times larger than that for  $m_x=2$  ( $\gamma(2) \sim \gamma(5)/2$ ) as estimated from the

green and blue curves in Figure 6a, using equation (1), the condition in which the  $m_x=2$  mode

reaches to a specific level *c* faster than the  $m_x=5 \mod (i.e., \Delta t(2) < \Delta t(5))$  can be described as,

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$$\frac{\ln\left[\frac{c}{b_0}\right] - |s| \cdot \ln\left(\frac{5}{2}\right)}{\frac{\gamma(5)}{2}} < \frac{\ln\left[\frac{c}{b_0}\right]}{\gamma(5)}.$$

283 This relation can be rewritten as,

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$$|\mathbf{s}| > \frac{\ln\left[\frac{c}{b_0}\right]}{2\ln\left(\frac{5}{2}\right)} \sim \ln\left[\frac{c}{b_0}\right]/2.$$
(2)

For runs with  $a_0 = \delta B_0(m_x = 4) = 0.01 B_0$  (runs-B, D and E), the initial amplitude for  $m_x = 5$  (i.e.,  $b_0$ ) 285 would be close for all runs. As seen in Figures 6b and 6d, the amplitude of the fluctuations for 286  $m_x=5$  for these runs is enhanced to a level around  $b_0^2 \sim 10^{-7} - 10^{-6} V_A^2$  in the initial phase. From 287 equation (2), the threshold for the  $m_x=2$  mode to reach a certain level  $c^2 \sim 10^{-5} - 10^{-4} V_A^2$  (shaded 288 levels in Figure 6) faster than the theoretical fastest  $m_x=5$  mode can be predicted as  $|s|\sim1$ . Indeed, 289 as shown in Figure 7a, the  $m_x=2$  mode exceeds the  $m_x=5$  mode for s=-5/3, both  $m_x=2$  and 5 290 modes reach the shaded level almost at the same time for s=-1, and the  $m_x=5$  mode exceeds the 291  $m_x=2$  mode for s=-1/3. In addition, for runs with  $a_0=\delta B_0(m_x=4)=0.001B_0$  (runs-C and G) (i.e., 292 runs in which  $b_0$  is set to be one-order of magnitude smaller than the above runs), from equation 293 (2) the threshold can be predicted as  $|s| \sim 2.5$ . And indeed, as shown in Figure 7b, the m<sub>x</sub>=2 mode 294 295 exceeds the  $m_x=5$  mode for s=-11/3, both modes reach the shaded level almost at the same time for s=-8/3, and the  $m_x=5$  mode exceeds the  $m_x=2$  mode for for s=-5/3. These results support the 296 adequacy of the prediction from equation (2). 297

Notice that as seen in Figure 6, once a mode reaches the level of  $c^2 \sim 10^{-5} - 10^{-4} V_A^2$  first, this mode causes the subsequent inverse cascade - i.e., the modes shorter than this mode can never grow dominantly. Thus, equation (2) suggested that when the amplitude level and the spectral index of the initial fluctuations are sufficiently high and small, respectively, the modes whose wavelengths are longer than the theoretical fastest growing mode can grow more dominantly than the theoretical fastest mode. This may explain the observational features of the KH waves at the magnetopause, whose wavelengths tend to be longer than the expected
wavelength of the theoretical fastest growing mode calculated from the typical thickness of the
magnetopause [Hasegawa et al., 2009]. This point will be discussed in section 4.2.

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### 3.3 Spectral evolution

Figure 8a shows averaged 1-D spectra of  $B_v$  (~B<sub>1</sub>) near the center of the vortex layer 309 over t=0-50  $\Omega_i^{-1}$  (initial phase), 100-150  $\Omega_i^{-1}$  (onset phase), 150-200  $\Omega_i^{-1}$  (early linear phase), 310 200-250  $\Omega_i^{-1}$  (later linear phase), 250-300  $\Omega_i^{-1}$  (early non-linear phase), 500-550  $\Omega_i^{-1}$  (later non-311 linear phase), and 750-800  $\Omega_i^{-1}$  (final merging phase) for  $\delta B_0=0$ . After the onset of the KH 312 instability, the power near the theoretical fastest growing KH modes ( $m_x=4-6$ ) is enhanced (see 313 the green to magenta curves). In the early non-linear phase, the power in smaller scale (sub-ion 314 scale) modes starts being enhanced due to the formation of the small-scale islands by the VIR 315 316 (see the cyan curve). A similar power enhancement at sub-ion scales is also seen in recent kinetic simulations of collisionless plasma turbulence in which reconnection occurs at multiple-points 317 318 and forms small-scale islands [Franci et al., 2017]. Notice that in this early non-linear phase, the spectrum at MHD scales has not well matured and its spectral index is still near -8/3 as also seen 319 320 in the past simulation [Nakamura et al., 2017a]. In the later phases, well-matured turbulent spectra with an index close to -5/3 at MHD scales and steeper ones with the minimum index 321 close to -4 at sub-ion scales appear as also seen in past simulations [Daughton et al., 2014; 322 Nakamura et al., 2017a]. 323

324 For all of the finite  $\delta B_0$  cases (Figures 8b-d), the spectral indexes in the well-matured phases are close to the ones for  $\delta B_0=0$  –i.e., ~-5/3 at MHD scales and ~4 or larger at sub-ion 325 scales (see yellow and black lines). However, for finite  $\delta B_0$ , not only the MHD-scale modes but 326 even the sub-ion scale modes grow stronger and earlier than the  $\delta B_0=0$  case. In particular, for 327 runs-B and E ( $a_0=0.01B_0$ ), the well-matured turbulent spectra with spectral indexes close to -5/3328 at MHD scales form even before the early non-linear phase (before t~250  $\Omega_i^{-1}$ ) (see magenta 329 curves in Figures 8b and 8d). For run-C ( $a_0=0.001B_0$ ), the spectral evolution is similar to the one 330 for  $\delta B_0 = 0$ , although the amplitudes for the MHD-scale modes are slightly stronger especially in 331 the linear phases (compare Figures 8a and 8c). Notice that for run-B (Figure 8b), the spectral 332

index in the early and later non-linear phases (t>250  $\Omega_i^{-1}$ ) is close to -5/3 at MHD scales, -3 at 333 sub-ion scale near  $k_x \sim 30-100$  and -4 at smaller scales (see cyan, yellow and black curves in 334 Figure 8b). This tendency is reasonably consistent with the MMS observations as will be 335 discussed in section 4.2. Notice also that these spectral features for finite  $\delta B_0$  especially at MHD 336 and sub-ion scales are not significantly affected by m<sub>i</sub>/m<sub>e</sub> as seen in Figure 9a which shows the 337 spectra for the case with the same setting as run-B (Figure 8b) but for  $m_i/m_e=100$  and the 338 339 corresponding two times smaller grid spacing. However, at sub-ion and smaller scales, the transition range in the spectra where the index is about -3 is somewhat wider for  $m_i/m_e=100$ 340 (compare red and blue curves in Figure 9b). This indicates that the transition from -3 to -4 slopes 341 could be affected by electron dynamics. The spectrum formation processes at the sub-ion to 342 electron scales will be investigated in more detail in future. 343

Figure 10 shows the spectra of  $|\mathbf{B}|$  and  $B_v$  (~ $B_\perp$ ) for run-A (top) and B (bottom). For both 344 runs, there is a clear tendency that the fluctuations develop dominantly in the perpendicular 345 direction in both MHD and sub-ion scales. The difference between  $|\mathbf{B}|$  and  $B_v$  at both scales 346 increases with time, and the power of  $B_y$  becomes more than one order larger than that of  $|\mathbf{B}|$  at 347 both scales in the well-matured phases. This is because both the KH instability at MHD-scales 348 and the subsequent VIR at sub-ion scales evolve mainly in the perpendicular direction. This 349 tendency is also reasonably consistent with past spacecraft observations of the KH waves at the 350 351 magnetopause as will be discussed in Section 4.2.

352

# **4. Summary and Discussion**

354 4.1 Summary

We have performed a series of 2-D fully kinetic simulations of the MMS event on 8 September 2015, in which the non-linear KH waves and clear signatures of the vortex-induced reconnection (VIR) were observed [Eriksson et al., 2016a; Nakamura et al., 2017a,b], newly considering initially-fluctuating magnetic field. The main results of this paper are summarized as follows:

- 1. For the initial field fluctuations, we add an ensemble of  $k_x$  modes with different 360 amplitudes to form a power-law spectrum. When the initial amplitude of a mode exceeds 361 the PIC noise level, the mode starts growing from the level of the initial perturbation. 362 2. In consequence of the competition among modes having different initial amplitudes, the 363 longer wavelength modes can grow dominantly, as the amplitude level and spectral index 364 of the initial fluctuations become higher and smaller, respectively. 365 3. For finite  $\delta B_0$ , the growth time-scale of the KH instability is small compared to the no 366  $\delta B_0$  case, leading to a more efficient plasma mixing within the vortex layer. 367 4. For large  $\delta B_0$ , the power of spectra is enhanced especially for the  $B_{\perp}$  component over 368 sub-ion to MHD scales, and the well-matured spectrum with a -5/3 index can be seen 369 even in the early non-linear growth phase of the KH instability.
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### 4.2 Comparison with observations

373 Figure 11 shows averaged power spectra of  $|\mathbf{B}|$  and  $B_{\perp}$  in the MMS observations on 8 September 2015 over the KH wave intervals and the adjacent magnetosheath interval (just after 374 the wave interval). The wave intervals employed here (Figures 11a and 11b) are the same as the 375 ones employed in Stawarz et al. [2016]. The magnetosheath interval is nearly the same as the one 376 used in the simulations of this event to obtain the initial simulation parameters on the 377 magnetosheath side in the previous (Nakamura et al. [2017a,b]) and present simulations. The 378 averaged magnetic field strengths of the wave and magnetosheath intervals are 70 nT and 75 nT, 379 respectively. The turbulent spectrum with an index close to -5/3 at MHD scales was observed 380 during the wave interval (Figure 11a), which was not seen in the early non-linear phase in the 381 past and present simulations with  $\delta B_0 = 0$  (see the cyan curve in Figure 8a and Figure 2 in 382 Nakamura et al. [2017a]). The power of the observed spectra for B<sub>1</sub> at both MHD and sub-ion 383 scales was more than one-order larger than that for  $|\mathbf{B}|$  (Figure 11b). The observed spectral index 384 for  $B_1$  at sub-ion scales (~1-100 Hz) is somewhat larger than -4 (close to -3), while that at 385 smaller-scales (above ~100 Hz) is close to -4. These points are reasonably consistent with the 386 present simulations for run-B in the early non-linear phase (see cyan curves in Figures 10c and 387 10d). 388

The turbulent spectra were also observed during the adjacent magnetosheath interval of 389 this MMS event (Figures 11a and 11b), as reported in past magnetosheath observations [e.g., 390 Alexandrova et al. 2008; Roberts et al., 2018]. A similar spectrum (the slope and the relative 391 amplitudes considering the background field strengths) was also observed during the adjacent 392 boundary layer interval of this event (not shown). In the simulation for run-B, the power of the 393  $B_v (\sim B_1)$  fluctuations near the center of the vortex layer is one or two order larger than that in the 394 background region over ion-scales to MHD scales (compare the cyan curve in Figure 10c and the 395 396 blue solid curve in Figure 1). Although the observed spectra in the magnetosheath (black curves in Figure 11) is not perfectly reproduced in the simulation, this spectral feature for run-B is 397 reasonably consistent with the observed relative enhancement of the B<sub>1</sub> fluctuations in the wave 398 interval compared to the magnetosheath interval in this MMS event (see Figures 11b). These 399 consistencies on the spectral features between the simulation for run-B and the MMS 400 observations suggest that the effects of fluctuating magnetic field on the vortex layer as seen in 401 the present simulations for large  $\delta B_0$  may really occur during this MMS event. 402

Figure 12 shows averaged power spectra of  $|\mathbf{B}|$  and  $B_{\perp}$  in the Geotail-Cluster conjunctive 403 observation event on 20 November 2001, which was reported in Hasegawa et al. [2009], over the 404 KH wave interval for Geotail (19:20-19:35 UT) and Cluster (19:20-19:35 UT) and the adjacent 405 magnetosheath interval for Geotail (19:00-19:15 UT). In this event, Geotail, located at the 406 postnoon (~15MLT) magnetopause, and Cluster, located at the dusk-flank (~19MLT) 407 magnetopause, simultaneously observed quasi-periodic KH waves. Hasegawa et al. [2009] 408 suggested that the growth phase of the observed KH waves at the Cluster location was in a range 409 of the non-linear phase, while that at the Geotail location was in a much earlier phase (probably, 410 411 onset to linear phases). The averaged magnetic field strengths of the wave and magnetosheath intervals are 40.9 nT (Geotail, wave), 20.6 nT (Cluster, wave) and 24,4 nT (Geotail, sheath), 412 respectively. Considering these field strengths during the observed intervals, the relative 413 amplitude of the B<sub>1</sub> spectrum observed by Geotail during the KH wave intervals compared to 414 415 that during the magnetosheath interval was about one order larger than the relative amplitude of the  $|\mathbf{B}|$  spectrum (see Figure 12a and 12c) especially at the MHD scales, as seen in the simulation 416 for run-B in the linear phase (compare red and blue curves in Figures 10c and 10d). This 417 indicates that the B<sub>1</sub> fluctuations were enhanced by the KH instability, as also seen in the above 418 MMS event. In addition, considering the background field strengths (i.e., normalized by the field 419

strengths), the relative B<sub>1</sub> amplitude during the wave interval especially at the MHD scales at 420 the Cluster location was about five times larger than that at the Geotail location (see Figure 12d). 421 This indicates that the field fluctuations within the vortex layer were additionally enhanced as the 422 423 KH instability traveled and evolved along the magnetopause as seen in the present simulations. Furthermore, while the observed spectral index for  $B_{\perp}$  at sub-ion scales at the Geotail location 424 was somewhat larger than -4, that at the Cluster locations was close to -4 (see Figure 12d). These 425 spectral features are roughly consistent with the present simulations (see Figure 10c), although 426 427 the relative fluctuations in the magnetosheath compared to the KH wave intervals in this Geotail-Cluster event are somewhat stronger than those in the above MMS event as well as the present 428 simulations. 429

In addition, in this Geotail-Cluster event, Hasegawa et al. [2009] reported that the 430 observed wavelength of the KH waves (>4 $\times 10^4$  km) was a few times longer than the one for the 431 theoretical fastest growing KH mode calculated from the thickness of the velocity shear layer 432  $(\sim 10^3 \text{ km})$ . This point is also reasonably consistent with the present simulation for run-B, in 433 which the  $m_x=2$  mode dominantly grows first and cascades to the  $m_x=1$  mode, and consequently 434 the theoretical fastest growing  $m_x=4-5$  mode never exceeds these longer wavelength modes. All 435 of these consistencies between the present simulation for finite  $\delta B_0$  (especially for run-B) and the 436 observations naturally suggest that the magnetic field fluctuations as observed in the 437 magnetosheath and boundary layer may really affect the subsequent evolution of the KH 438 instability at the magnetopause. 439

Note that the simulations performed in this study setup the constant spectral indices in the 440 initial state, despite the observed spectra near the magnetopause commonly feature smaller 441 442 indices at sub-ion and smaller scales as seen in Figures 11 and 12. This may lead to a stronger evolution of the small-scale modes than the case with a non-constant index. Thus, to more 443 quantitatively comparae the simulations with the observations, more realistic initial fluctuations 444 need to be setup by considering the nonconstant spectral index, as well as the ky and kz modes, 445 446 which are also not considered in this study. In addition, although this paper sets up the initial fluctuations only in the magnetic field, strong fluctuations in plasma parameters such as flow 447 velocity have also been frequently observed in the magnetosheath [e.g., Plaschke et al., 2019]. 448 Although the present simulations showed that initial magnetic field fluctuations immediately 449

enhance the velocity fluctuations (see section 3.2), setting up the initial fluctuations even in the 450 flow velocities as well as the other parameters such as density may also be important to discuss 451 the effects of the background turbulence more quantitatively. Furthermore, since spectral 452 features of the background turbulence, such as slopes and amplitudes at each scale, are variable 453 depending on various factors such as the plasma  $\beta$  [e.g., Alexandrova et al., 2008], to more 454 comprehensively understand how strongly the background turbulence affects the KH instability 455 at the magnetopause, it will be needed to statistically analyze more events at different locations 456 under different conditions. Based on the initial studies shown in this paper, such more practical 457 approaches in both simulation and observation may lead to a more realistic and quantitative 458 understanding of the roles of the KH instability at the magnetopause. 459

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## 4.3 Some other remarks on future work

Although this study treats only 2-D simulations, recent 3-D fully kinetic simulations of 462 the KH instability showed the importance of the 3-D effects especially on the formation of the 463 secondary flux ropes [Nakamura et al., 2013; 2017a,b]. These simulations demonstrated that in 464 3-D, the VIR can be induced over a range of oblique angles, leading to a more turbulent 465 formation of the flux ropes within the vortex layer. Recent 3-D two-fluid simulations also 466 showed that when considering a more realistic magnetopause-like situation in which the low-467 latitude unstable region of the KH instability is sandwiched between the high-latitude stable 468 regions, the VIR can be induced even in the mid-latitude region where the growth of the KH 469 instability is much weaker than the low-latitude region [e.g., Fadanelli et al., 2018]. Moreover, 470 recent 3-D fully kinetic simulations of the magnetopause demonstrated that the density jump 471 472 across the magnetopause can potentially cause electromagnetic turbulence induced by the lowerhybrid drift instability (LHDI) [Price et al., 2016; Le et al., 2017]. Such an LHDI turbulence at 473 the magnetopause would also be coupled with the KH instability under certain conditions. Indeed, 474 intense LHD waves were recently observed by MMS within the KH instability at the 475 magnetopause [Tang et al., 2018]. Investigating the coupling of these 3-D effects with the 476 477 background turbulence is an important direction for future research.

478 As mentioned in section 3.3, the formation of spectral slopes including sub-ion and 479 smaller scales may be affected by the VIR and the resulting turbulent formation of the magnetic

islands within the vortex layer. Based on hybrid (ion kinetic and mass-less electron fluid) 480 simulations, Franci et al. [2017] suggested that the turbulent formation of small-scale islands can 481 produce an inverse cascade through the island merging in addition to the direct cascade through 482 the island formation. This two-way cascade leads to the formation of the spectrum with a ~-3 483 index at sub-ion scales, which is reasonably consistent with the present simulation as seen in 484 Figures 9b. Past fully kinetic simulations demonstrated that in the VIR process, reconnection is 485 first induced at compressed current layers whose thicknesses are of the order of de [Nakamura et 486 al., 2011; 2013], indicating that the two-way cascade as seen in Franci et al. [2017] would occur 487 even at electron scales in the fully kinetic regime. On the other hand, past theories suggested that 488 electron flow turbulence in the direction perpendicular to the magnetic field can cause additional 489 energy cascade from MHD to electron scales and form the spectrum with a -4 or smaller index at 490 electron scales [e.g., Narita, 2016]. The secondary turbulence within the vortex layer may drive 491 additional reconnection as seen in the magnetosheath turbulence [e.g., Retinò et al. 2007; 492 Yordanova et al. 2016; Vörös et al. 2017; Phan et al. 2018; Stawarz et al. 2019], which may also 493 contribute to energy dissipation and plasma mixing and/or influence the small-scale non-linear 494 495 dynamics. Investigating such sub-ion to electron scale physics related to the KH instability and understanding which processes dominantly transfer energies at these scales are also an important 496 497 direction for future research.

Finally, since it is known that the field and flow fluctuations tend to be stronger in the 498 dawnside magnetosheath (i.e., in the downstream of the quasi-parallel shock for the solar wind 499 Parker spiral), the effects of turbulent fluctuations on the KH instability as seen in the present 500 simulations may be more important at the dawnside magnetopause [e.g., Luhmann et al., 1986; 501 Plaschke et al., 2019]. For example, Moore et al. [2016] showed an observation event of the KH 502 waves at the dawn-flank magnetopause in which the cold ions were effectively heated by ion-503 scale waves within the KH waves. Although it is difficult to treat these global pictures in the 504 fully kinetic regime, global hybrid (fluid-electron and kinetic-ions) simulations, which recently 505 began to be applied for the whole Earth's magnetosphere [Palmroth et al., 2018 and references 506 therein], may provide important support for treating such global physics of the magnetosheath 507 turbulence and its relation to the KH instability at the magnetopause. 508

### 510 6. Conclusions

We have performed a series of 2-D fully kinetic simulations of an MMS observation 511 event on 8 September 2015, in which the non-linear KH waves were observed at the Earth's 512 513 magnetopause, considering the pre-existing turbulent magnetic field fluctuations. This is the first numerical challenge to investigate the effects of fluctuating magnetic field on the growth of the 514 KH instability at the magnetopause in the fully kinetic regime. Here we setup the initial 515 fluctuations by adding an ensemble of k<sub>x</sub> modes to the magnetic field and perform runs changing 516 517 the amplitude level and the spectral index of the fluctuations. The results demonstrate (i) that 518 when the amplitude level is sufficiently high, the initial fluctuations can cause a faster evolution of the KH instability, leading to a more efficient plasma mixing within the vortex layer, and (ii) 519 that longer wavelength modes grow more dominantly as the spectral index becomes smaller with 520 negative sign. For the cases with large amplitude of the initial fluctuations, the well-matured 521 522 spectrum with a -5/3 index appears even in the early non-linear growth phase of the KH instability. These simulation results are reasonably consistent with the observations in this MMS 523 524 event and a past Geotail-Cluster conjunction event in which Geotail and Cluster observed the KH waves at the postnoon and dusk-flank magnetopause, respectively. Based on these initial results, 525 future researches with more realistic approaches, for example, by setting more realistic spectral 526 signatures of the initial fluctuations may lead to a more quantitative understanding of the effects 527 of the background turbulence on the evolution of the sheared boundary at the magnetopause. 528

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- 542
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Table-1. *s* (the spectral index of  $\delta B_0^2$ ) and  $a_0$  (= $\delta B_0$  for m<sub>x</sub>=4) employed in the simulations shown in this paper.

Run	А	В	С	D	Е	F	G
S	N/A	-5/3	-5/3	-1	-1/3	-11/8	-8/3
$a_0/\mathbf{B}_0$	0	0.01	0.001	0.01	0.01	0.001	0.001





- 707 112.5±14.6 d<sub>i</sub>) for run-B ( $\delta B_0^2 \propto k_x^{-5/3}$ ,  $\delta B_0(m_x=4)=0.01B_0$ ) and run-C ( $\delta B_0^2 \propto k_x^{-5/3}$ ,
- 708  $\delta B_0(m_x=4)=0.001B_0$  at t=0 (initial) and averaged over t=0-50  $\Omega_i^{-1}$ , 100-150  $\Omega_i^{-1}$ , 200-250  $\Omega_i^{-1}$ ,
- 400-450  $\Omega_i^{-1}$ , 800-850  $\Omega_i^{-1}$ . The vertical lines in Figure 1b indicate the wavelengths for  $k_{\perp}\rho_i = 1$
- 710 and  $k_{\perp}d_i = 1$ .



Figure 2. Time evolution from the linear (a,e) to the late non-linear (d,h) growth phase of the KH instability of 2-D contours of the ion mixing fraction  $F_i$  for run-A ( $\delta B_0=0$ ) (a-d) and run-B ( $\delta B_0^2 \propto k_x^{-5/3}$ ,  $\delta B_0(m_x=4)=0.01B_0$ ) (e-h). Grey curves show in-plane magnetic field lines.



Figure 3. The same as Figure 2, but for run-C  $(\delta B_0^2 \propto k_x^{-5/3}, \delta B_0(m_x=4)=0.001B_0)$  (a-d) and run-E  $(\delta B_0^2 \propto k_x^{-1/3}, \delta B_0(m_x=4)=0.01B_0)$  (e-h).



Figure 4. Time evolution of the averaged thickness  $\langle L_{mix} \rangle$  of the mixing layer (defined by the

region with  $|F_i| < 0.9$  in the y-direction for runs-A (red), B (green), C (blue) and E (magenta).



Figure 5. Time evolution of the 1-D power spectra ( $k_x$ ) of U<sub>iy</sub> modes around the center of the boundary ( $y=0\pm14.6d_i$ ) for runs-A (a), B (b), C (c) and E (d).



Figure 6. (a) Time evolution of  $U_{iy}$  modes at the center of the boundary (y=0) for  $m_x=10$  (red), 5 (green), 2 (blue) and 1 (magenta) modes for runs-A (a), B (b), C (c) and E (d).



Figure 7. The same as Figure 6 but for  $m_x=5$  (solid) and 2 (dashed) modes (a) with

732  $\delta B_0(m_x=4)=0.01B_0$  for runs-B ( $\delta B_0^2 \propto k_x^{-5/3}$ ) (red), D ( $\delta B_0^2 \propto k_x^{-1}$ ) (green), and E ( $\delta B_0^2 \propto k_x^{-1/3}$ )

- 733 (blue), and (b) with  $\delta B_0(m_x=4)=0.01B_0$  for runs-F ( $\delta B_0^2 \propto k_x^{-11/3}$ ) (red), G ( $\delta B_0^2 \propto k_x^{-8/3}$ ) (green),
- 734 and C  $(\delta B_0^2 \propto k_x^{-5/3})$  (blue).



Figure 8. 1-D power spectra ( $k_x$ ) of  $B_y$  averaged over t=0-50  $\Omega_i^{-1}$ , 100-150  $\Omega_i^{-1}$ , 150-200  $\Omega_i^{-1}$ , 200-250  $\Omega_i^{-1}$ , 250-300  $\Omega_i^{-1}$ , 500-550  $\Omega_i^{-1}$ , and 750-800  $\Omega_i^{-1}$ , around the center of the boundary (y=0±14.6 d<sub>i</sub>) for runs-A (a), B (b), C (c) and E (d). The vertical lines indicate the wavelengths for  $k_\perp \rho_i = 1$  and  $k_\perp d_i = 1$ .



Figure 9. (a) The same as Figure 8b but for  $m_i/m_e=100$ . (b) Spectra of  $B_y$  averaged over t=250-

744 300  $\Omega_i^{-1}$  (dashed) and 750-800  $\Omega_i^{-1}$  (solid) for  $m_i/m_e=25$  (red) and  $m_i/m_e=100$  (blue). The vertical

colored lines indicate the wavelengths for  $k_{\perp}d_e = 1$  and  $k_{\perp}d_{e2} = 1$  for  $m_i/m_e=25$  (red) and

 $m_i/m_e=100$  (blue), where  $d_e$  and  $d_{e2}$  are the electron inertial lengths based on  $n_1=n_0$  and  $n_2$ .



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Figure 10. 1-D power spectra ( $k_x$ ) for run-A (top) and run-B (bottom) of  $B_y$  (left) and  $|\mathbf{B}|$  (right)

and  $B_y$  (bottom) for the same intervals and area as Figure 8. Figures 10a and 10c are the same as Figures 8a and 8b.



Figure 11. Averaged power spectra (Hz) of  $|\mathbf{B}|$  (top) and the perpendicular component of the 752 magnetic field  $B_{\perp}$  (bottom) over the four MMS spacecraft and 38 selected intervals on 8 753 September 2015 during which MMS observed quasi-periodic KH waves for ~80 minutes at the 754 postnoon (~16 MLT) magnetopause (see Stawarz et al. [2016] for more details about the 755 techniques to obtain the averaged spectra of this event). The black curves show averaged spectra 756 during the magnetosheath interval (11:35:00-11:50:20 UT). Here the magnetic field data during 757 the KH interval are obtained from the Fluxgate Magnetometers (FGM) [Russell et al., 2016] and 758 Search-Coil Magnetometer (SCM) [Le Contel et al., 2016], while the data during the 759 760 magnetosheath interval are from the FGM. The vertical lines indicate the ion cyclotron frequencies  $f_{ci}$  and the wavelengths for  $k_{\perp}\rho_i = 1$  and  $k_{\perp}d_i = 1$  under the assumptions of Taylor 761 hypothesis  $(2\pi f = k_{\perp}V_{\perp})$ . 762 763



Figure 12. Averaged power spectra (Hz) of  $|\mathbf{B}|$  (top) and  $B_{\perp}$  (bottom) by Geotail (blue) and 765 Cluster-1 (red) over 19:20-1935 UT on 20 November 2001, during which both Geotail and 766 Cluster observed quasi-periodic KH waves at the postnoon (~15 MLT) and dusk-flank (~19 767 768 MLT) magnetopauses, respectively [Hasegawa et al., 2009]. The black curves show averaged spectra during the magnetosheath interval (19:00-1915 UT) observed by Geotail. Here the 769 magnetic field data are obtained from the FGM for Cluster [Balogh et al., 2016] and the 770 magnetic field experiment (MGF) for Geotail. The vertical lines indicate the ion cyclotron 771 772 frequencies  $f_{ci}$  and the wavelengths for  $k_{\perp}\rho_i = 1$  and  $k_{\perp}d_i = 1$  under the assumptions of Taylor hypothesis  $(2\pi f = k_{\perp}V_{\perp})$ . 773

Figure 1.



Figure 2.



Figure 3.



Figure 4.



Figure 5.



Figure 6.



Figure 7.



Figure 8.



Figure 9.



Figure 10.



Figure 11.



Figure 12.

