Assessment of Covariance Estimation through Least Squares Collocation over Iran

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Abstract

In our previous work [Ramouz et al. 2018], during the gravity field determination via Least Squares Collocation (LSC) in Iran, it was detected that localizing covariance modeling shows better performance than using one uniform covariance for all the under investigation regions. Now the question is which criteria should be used for dividing the region into subareas for localization the covariance estimation? Tscherning et al. 1994 stated that data distribution could significantly affect the covariance estimation and consequently the LSC gravity modeling. As Iran has a rough topography and at the same time suffers from lack of a good coverage and homogenous terrestrial gravity network, covariance analysis in this area is not a trivial task. Four local case studies with different roughness and data distributions but with the same window size were selected. In each case study and based on Remove – Restore technique, the global and topographic parts of the gravity signal were removed from the observations. To do so, global gravity model EIGEN-6C4 up to d/o 360 and RTM method with the topographic information supplied by SRTM 1 arc-second height model, were used respectively. After that, residual gravity anomalies went through analytical covariance estimation by make use of Tscherning – Rapp 1974 covariance model. Indeed, covariance estimation in LSC method consists of two steps: calculation of an empirical covariance function from the residual gravity anomalies, and fitting an analytical covariance model to it. In this study, we focus on the considerations about data and its distribution which must be taken into account during empirical and analytical covariance determination. In case of not well-distributed input data, excavating analytical covariance model parameters is a challenging task. In some cases, this sensitivity causes difficulty even in choosing initial values for inverse adjustment of these parameters, which improper initial values lead to wrong parameters selections. Also, the distribution of data in each case study was manipulated to analyze its influence on the covariance estimation. To make an assessment, in each case study, the residual gravity anomalies were split into two datasets; first as observations input for LSC, and the second, as control points to evaluate the accuracy of the LSC gravity modeling and the covariance estimation. Then the interdependency and effect of Tscherning – Rapp covariance model parameters on the covariance estimation were investigated in each case study. Evaluation of the results in the case studies shows that the accuracy of the gravity modeling, directly dependent on the distribution of the data and the roughness of the topography, among other parameters. Finally, enhancing the covariance estimation based on presented approach, lead to about 10% improvement of the accuracy in terms of STD through LSC gravity modeling.

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Introduction

To implement Least Squares Collocation (LSC), usually Remove–Compute–Restore (RCR) technique is used. In RCR, first the systematic parts of the gravity signal related to the global and topographical effects were removed, then restored after the LSC estimation [Sanso and Sideris, 2013]. One of the most critical task in LSC gravity field modeling is the Covariance (COV) determination. Tscherning and Rapp [1974] introduced a harmonic 3D COV model (TR1974) with

$$\left[\frac{R_B^2}{r_P r_Q}\right]^{\ell+1} \frac{a}{(\ell-1)(\ell-2)(\ell+4)}$$

as degree variance, where r_P and r_Q are the radii of the Earth in points *P* and *Q*. Indeed, TR1974 fit an analytical COV model, to the empirical one which is extracted from the local observations using the three unknown variables α and *a* (the former related to the GGM error, the latter to the residual signal at higher degrees), and the Bjerhammer radius R_B[Moritz, 1980].

The quality of COV determination is sensitive to the data distribution through the case study. It is expected that in regions with dense and well distributed data, COV determination lead to better gravity modeling via LSC. In Ramouz et al., [2019], to localize the COV determination procedure, the region divided into four approximately equal parts. The heterogeneity and the lack of data in some parts of the case study lie at the root of the simplicity in the region division of their work. This study is devoted to analyze the effect of data distribution on the quality of COV determination through LSC modeling. Also, the effect of the topography roughness of the region on the COV modeling is examined. In addition, using the trial and error method, the best TR1974 parameters for each region are studied.

Data and its reductions

For COV analysis, four regions by 2.5*3 arc-degree area and different characteristics were chosen in Iran (Fig. 1). First and third regions (R1 and R3) with approximately 5 arc-minute network resolution. Note that R1 has relatively smoother topography than R3 (see Table1). Second and forth regions (R2 and R4) with approximately 13 arc-minute network resolution. In this case, R2 has relatively rougher topography than R4. Used gravity data consist of terrestrial observations from zeroth, first, second, third order gravity networks and gravity observations from a first order precise leveling network has included [Ramouz et al, 2019].

Removing the global field and the topographic effects from the gravity data impressively smoothed the gravity signal (Fig. 2). In particular, removing the GGM effects reduced *STD* of the observed gravity data by about 38%. After removing topographic effects using the RTM technique, *STD* of the gravity data decreased down to 69% (Table 1).

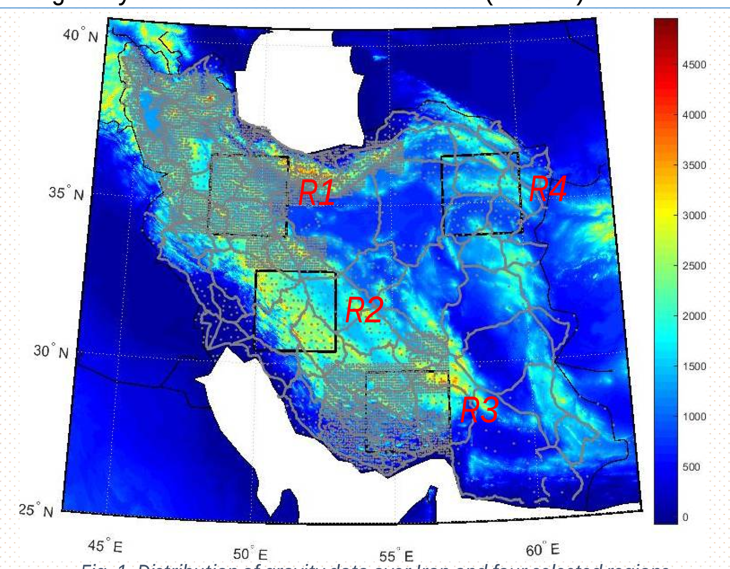

Fig. 1. Distribution of gravity data over Iran and four selected regions.

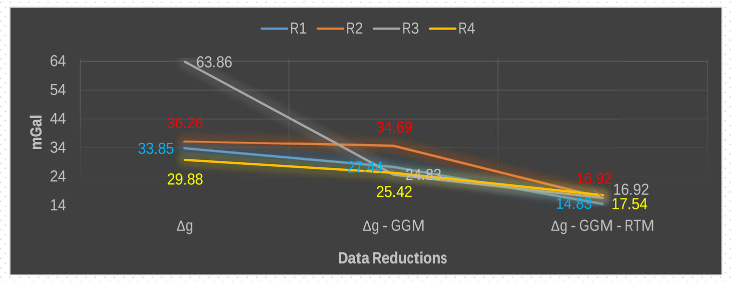

Fig 2. gravity anomalies before and after removing global (GGM) and topographic (RTM) effects (mGal) in each region

Table 1. Percentage of removing global and topographic effects on the STD of the gravity signal in each region(percentage)

Region	1		2		3		4	
Remove	mGal	Percent	mGal	Percent	mGal	Percent	mGal	Percent
Global	11.4	18.9	1.3	4.3	39	61.1	4.5	14.9
Topographic	12.6	46	17.7	51.2	7.9	31.9	7.9	31
Global + Topographic	24	56.2	19.1	53.3	46.9	73.5	12.3	41.3
Data Distribution	Dense		Sparse		Dense		Sparse	
Topography	Smooth		Rough		Rough		Smooth	

Covariance Analysis

To do the evaluation of the COV estimation, in each region the gravity anomalies were divided into two sub-sets; observations and control. For this reason, R1 and R3 was tiled to a set of 7*7 arc-min windows, and by the same manner R2 and R4 to 14*14 windows. Then, alternately these windows were classified as observations and control (Fig. 3). It should be noted that, at the edge of each region, control points excluded by a 15 arc-min strips.

Covariance Estimation

COV estimation was executed using TR1974 model and similar to Ramouz et al. [2019]. First, empirical COV was calculated using residual gravity anomalies. Then, after prediction of the TR1974 degree variance parameters, an analytical COV model was fitted to the empirical COV. In order to produce an empirical COV for a dataset, it is required to select an interval quantity (sample interval) which should be proportionate to the overall distribution of the data in the region. By sample interval in hand, the empirical COV will be computed and two parameters of the empirical COV, covariance at distance zero or variance (C_0) and the correlation distance (ξ) could be determined.

Refinement of gravity data distribution

Partly rough empirical COV in Fig. 4 and their modeled COV which did not fit adequately, encouraged us to check the effect of data distribution's refinement on the improvement of COV modeling. To that aim, data of R1 and R3 were smoothed by a 1.5, and R2 and R4 by a 2 arc-minute minimum-distance criterion to reduce the print of heavily linear crowded PLN observations (Fig. 5). As you can see from Fig. 6, this attempt could improve COV fitting, at least geometrically.

Assessment of covariance estimation

The effect of the distribution's refinement of the datasets was compared visually on the basis of the fitness between determined empirical and estimated analytical COV. Here, this refinement will also be evaluated statistically by accuracy assessment of the LSC output with the control points. For this purpose, LSC gravity field modeling must be implemented on the datasets. To execute the LSC procedure, beside the COV parameters, the observation and control subsets were used as the input and output of the LSC process respectively in each region. In Table 2, the results of the LSC modeling accuracy in regards to control data are shown for uniform, partial and local (before and after considering distribution's refinement) COV solutions. To estimate LSC uniform and partial solution, the related COV's parameters are obtained from Ramouz et al. [2019].

Table 2 illustrates that, although fitness between computed empirical and analytical COVs enhanced after distribution's refinement on the datasets, accuracy of the LSC modeling deteriorated in our case studies. Table 2 also shows that localization of the COV determination does not necessarily lead to improved LSC gravity modeling in any case study. The footprint of data distribution and topography roughness effects on the local gravity field modeling are detectable. One can see that the localized COV determination in R2 with relatively rough topography is successful, while seems not to work in R1 and R4. Furthermore, improvement in R2 with sparse data distribution is more than dense distributed R3. Actually, COV determination in sparse region is more sensitive than dense one. This phenomenon is obvious in Table 3, where the statistics of COV solutions' results in comparison with control points in each region are depicted. The variance of the Mean and STD in R2 and R4 are much more than R1 and R3. As it is showed in the last row of Table 3, the big part of these variations are stimmed from the local COV solutions.

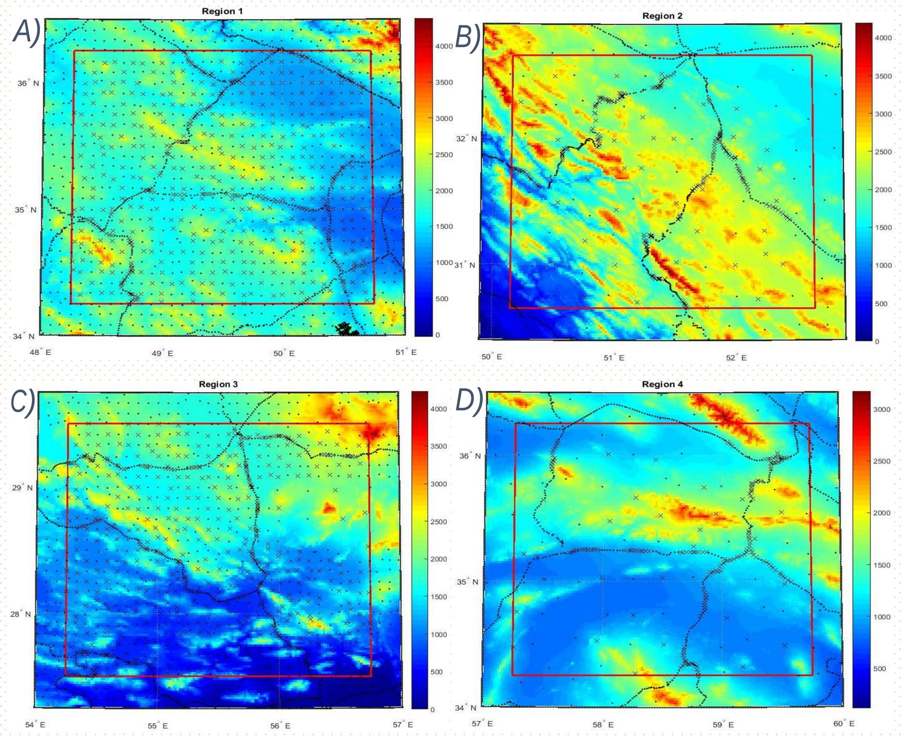

Fig. 3. Divided data into observations (dots) and controls (crosses) in A) R1, B) R2, C) R3 and D) R4.

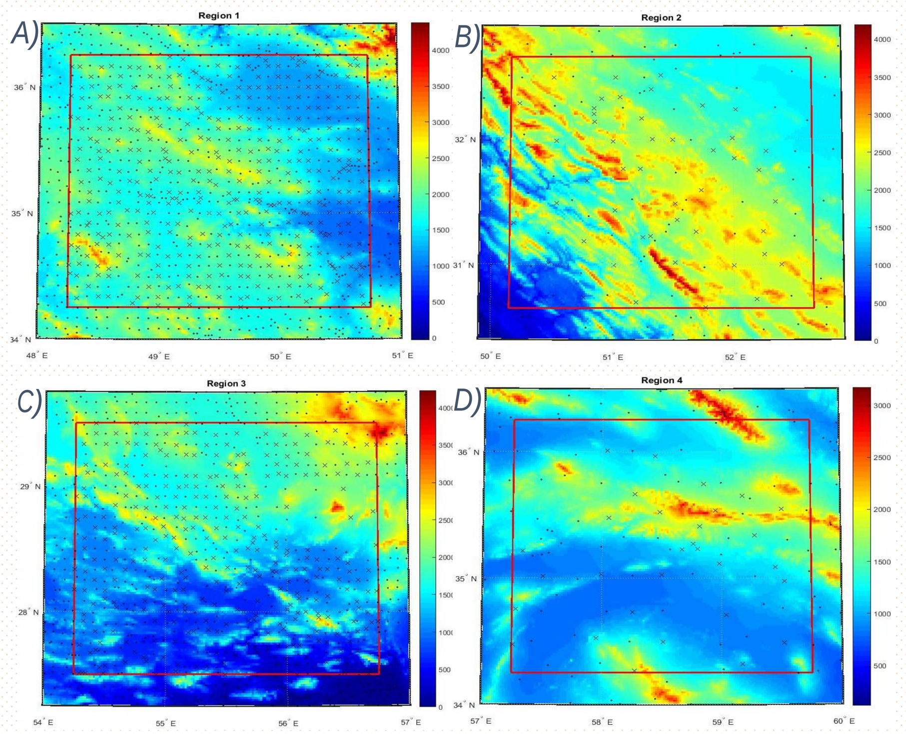

Fig. 5. Observations (dots) and controls (crosses) in A) R1, B) R2, C) R3 and D) R4 after refining the data.

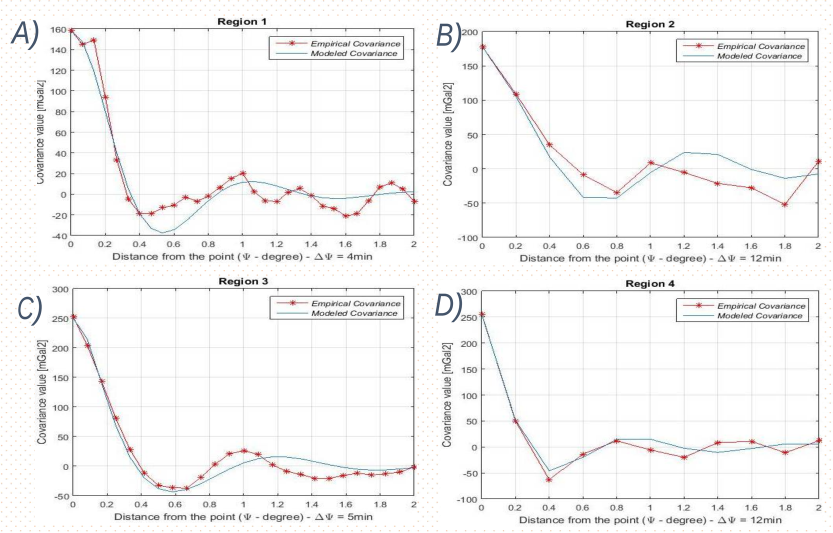

Fig. 4. Empirical and fitted COV for gravity data of A) R1, B) R2, C) R3 and D) R4.

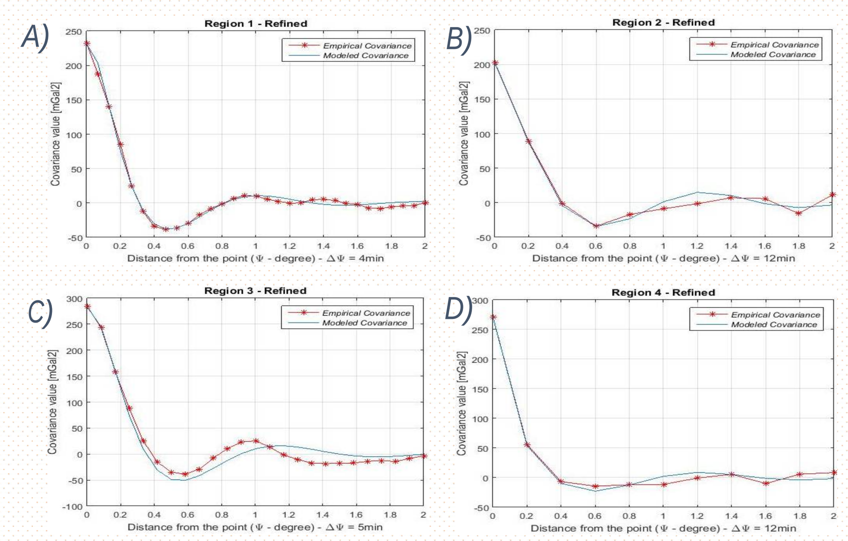

Fig. 6. Empirical and fitted COV of A) R1, B) R2, C) R3 and D) R4 after refining the data.

Conclusions and Future works

Relation between topography and data distribution on the data reductions during the COV estimation

As it is illustrated in Table 1, data reductions based on RCR technique in regions with denser distribution are more influential. In a way that in R1 and R3, averagely 65 percent of the gravity anomaly signal is reduced, while in R2 and R4 about 47 percent of the signal. Also, one can find a relation between the regions topography pattern and the data reductions. Between R1 and R3, reduction in R3 which has relatively rougher topography is more effective, as well as reduction in R2 between sparser distribution R2 and R4. It should be noted that density of the data distribution has more influence on the reductions than the topography roughness.

This study showed that the density of the data distribution in spite of topography roughness, has the same effect on the COV determination and LSC gravity modeling. That is to say, the accuracy of the LSC models in R1 and R3 is better than R2 and R4 in regards with control points. But, the topography roughness has a reverse effect on COV determination. In Table 2, between denser data distribution regions R1, and between sparser, R4 have better accuracy in comparison with control points. Both R1 and R4 have relatively smoother topography pattern.

Effect of refinement of data distribution through COV estimation on the LSC gravity modeling

Naturally, non-homogeneous data distribution over the regions (Fig. 2), led to rugged empirical COV functions like those in Fig. 3, and necessarily, the analytical COV function could not fit the empirical COV in the best way. By refining the data, the data distribution could be improved to obtain a more homogeneous one (Fig. 4), and consequently, get smoother empirical COV with better fitted analytical COV such as those in Fig. 5. In addition, the effect of data distribution refinement on COV determination on the LSC modeling was investigated and depicted that despite of visual analysis, refining the data distribution could not enhance the accuracy of LSC models in regards to the control points in the regions (Table 2).

Discussion and future works

This study showed that COV localization could be effective in regions with bad or sparse data distribution. Altogether, deriving the proper final output of analytical COV parameters is a challenging task. Although the sample interval for empirical COV and mean data spacing for analytical COV could be defined based on the region data distribution, finding the three final parameters of the COV model is quiet difficult.

Analyzing the quality of common used method for TR1974 COV model parameters and results of COV improvement on GNSS/Leveling control points are the suggested future works in this field.

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Table 2. Accuracy of the LSC gravity estimation based on uniform, partial and local COVs modeling in each region(mGal).

Region	1				2				3				4			
COV	Uni	Part	Local: Not ref	Local: Ref	Uni	Part	Local: Not ref	Local: Ref	Uni	Part	Local: Not ref	Local: Ref	Uni	Part	Local: Not ref	Local: Ref
Min	-18.3	-18.1	-5.32	-20.0	-29.7	-27.7	-28.6	-27.0	-50.8	-46.4	-46.4	-47.0	-48.3	-51.3	-43.1	-33.5
Max	19.7	20.0	14.8	16.9	45.6	51.2	53.6	51.3	34.5	34.2	34.5	34.6	26.2	29.7	35.3	21.8
Mean	-0.6	-0.5	-0.07	-1.2	-2.7	-1.9	-1.2	-1.9	-2.1	-2.3	-2.1	-2.2	-2.2	-1.9	-3.7	-3.8
STD	4.38	4.37	4.50	4.65	12.59	12.29	11.72	12.46	6.68	6.54	6.56	6.70	9.91	9.71	10.27	11.27

Table 3. Variance of the mean and STD of the COV solutions (mGal) in each region, in addition to their data distribution and topography patterns.

Region		1	2	3	4
Data distribution		Dense	Sparse	Dense	Sparse
Topography		Smooth	Rough	Rough	Smooth
Variance of Mean		0.16	0.28	0.01	0.73
Variance of STD	All solutions	0.01	0.11	0.01	0.36
	Local solutions	0.01	0.14	0.00	0.25

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