Double-difference/slope tomography by a variational projection approach

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Abstract

Dense acquisition are more and more available in exploration and earthquake seismology. Tomographic approaches can now consider not only travel times but also the wavefront itself across the seismic network (Zhang and Thurber, 2003; Yuan et al., 2016). For dense controlled-source seismic experiments, double differences of travel times between receivers in a common-shot gather (resp between sources in a common-receiver gather) are estimated, namely the horizontal component of the slowness vector at source and receiver positions designed as slopes. These slopes associated with the two-way traveltimes are interpreted as a reflection/diffraction from a small reflector segment or diffractor are used in tomographic inversion (Lambaré, 2008; Tavakoli F. et al., 2017). Picking of locally-coherent events leads to dense volumetric dataset and hence higher-resolution tomographic results (Guillaume et al., 2008). The reflection setting introduces implicitly another class of unknowns which are scatterer positions. Resulting inverse problem is awkward due to the intrinsic coupling between velocities and scatterer positions. The first choice alternates positions and wavespeeds. The second performs the joint estimation of the two parameter classes. The third one relies on the projection of the scatterer positions subspace onto the wavespeed subspace leading to a reduced-space inversion. This reduced-space formulation can be implemented in the slope tomography using adjoint-state method. Two focusing equations, which depend on two observables among the three available ones (two-way traveltime and one slope in 2D), gives exact solutions of positions which are injected as constraints in the slope tomography (Chauris et al., 2002). These constraints explicitly enforce the positions in the velocity estimation problem, which reduces now to a mono-variate inverse problem by minimization of single-slope residuals, not yet used. 2D synthetic (see figure) and real data case studies show faster convergence toward more accurate minimizer achieved by this variable projection method compared to the alternated and joint strategies. This method, which can be extended to 3D configurations, draws also interesting perspective for the joint hypocenter-velocity inversion problem in earthquake seismology.

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I - Context

• **Stereotomography** (slope tomography) (Lambaré, 2008), a **velocity macro-model building** method, exploits the horizontal component of the slowness vector at source and receiver positions. The **two slopes** associated with the **two-way traveltimes** define a **locally coherent event** in the data volume associated with a scatterer in the image domain.

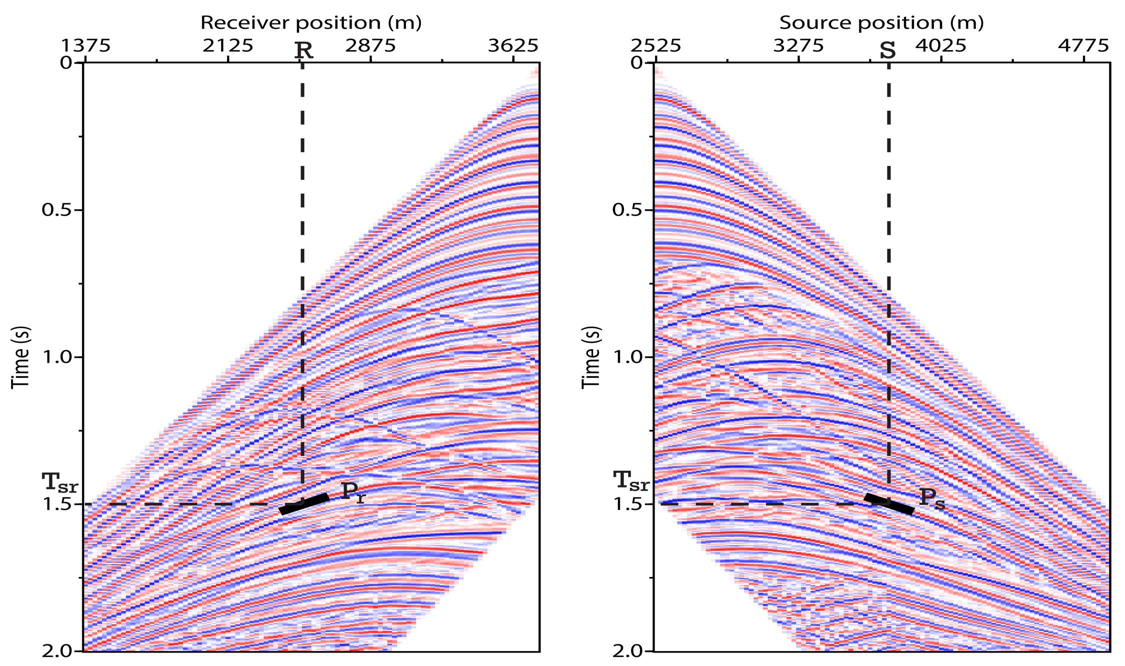

Figure 1: A locally coherent event picked in the data.

• **We address the issue of the ill-famed velocity-position coupling inherently present in reflection tomography. The strategy presented in this context draws perspectives to the analogous localization problem in earthquake seismology.**

II - Method

• We opt for the **matrix-free formulation** of slope tomography (AST) (Tavakoli F. et al., 2017) based on the **adjoint-state method** (vs. Fréchet derivatives) for the gradient computation. The forward problem is performed with **eikonal solvers** (vs. ray tracing).

• Commonly, the chosen optimization strategy aims at fitting all objective measures (**two-way traveltime and both slopes**) per scatterer, in search of **the velocity field and the scattering position jointly**.

• We propose a parsimonious formulation (PAST) that reduces the problem to fitting **one slope** in seek of **the velocity field** through a variational approach.

• **How?** An identified event in the data volume can be mapped in the image domain through a **kinematic migration** by means of the **focusing equations** (Chauris et al., 2002).

• **So what?** We elaborate on this relationship and how it is implemented in the form of **enforced physical constraints** under AST's framework and its implications on the **velocity-position coupling**.

III - Towards a velocity-position consistent formulation

In the proposed parsimonious approach we aim to solve the following minimization problem:

$$\min_{\mathbf{m}} J(\mathbf{m}) = \min_{\mathbf{m}} \sum_{s=1}^{N_{\mathbf{s}}} \sum_{r=1}^{N_r^s} \sum_{n_{s,r}=1}^{N_{\mathbf{n}}^{s,r}} \|(p_{s,n_{s,r}}(\mathbf{m}) - p^*_{s,n_{s,r}})\|^2,$$

where N_s / N_s^r / $N_{n_{s,r}}$ denotes the number of shots, receivers and events for a source/receiver pair (s,r). The symbol $*$ denotes the observed data. The predicted slope $p_{s,n_{s,r}}(\mathbf{m})$ depends on the model parameters through a nonlinear forward problem operator $\mathcal{F}$ which gathers the eikonal equation, the finite-difference approximation of slopes and the **focusing equations** (1) **and** (2) **(figure below)**.

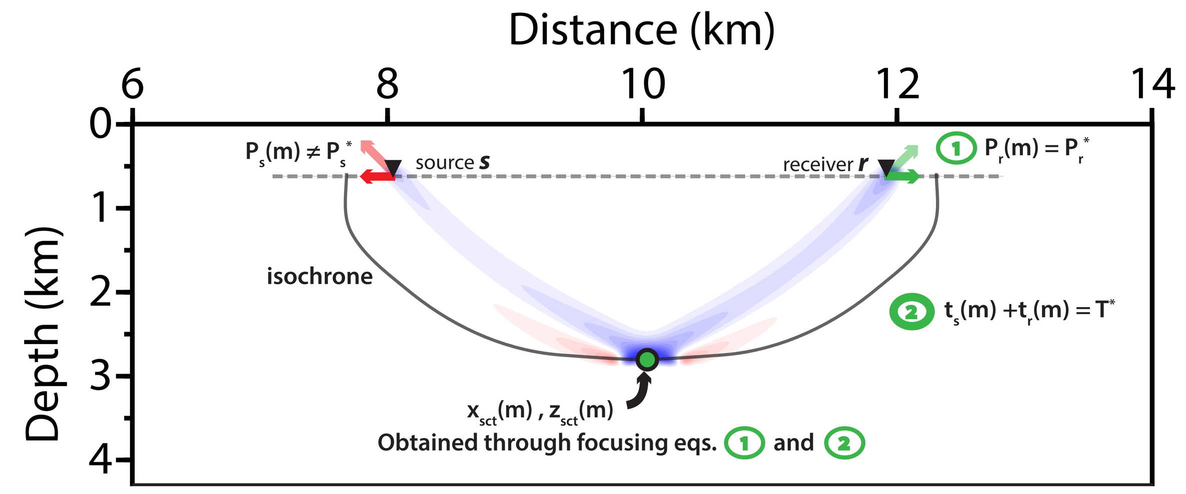

Figure 2: Focusing equations sketch superimposing a sensitivity kernel for a single scattering event.

We proceed under the reduced-space approach of the adjoint-state method (Plessix, 2006) for the gradient computation: $\mathcal{L}(\mathbf{m},\mathbf{u},\bar{\mathbf{u}}) = J(\mathbf{u}) - \left\langle \bar{\mathbf{u}}, \mathcal{F}(\mathbf{u},\mathbf{m}) \right\rangle$, where $\langle .,. \rangle$ denotes the inner product, $\mathbf{u}$ gathers the state variables, $\bar{\mathbf{u}}$ the adjoint-state variables.

The projection of the scatterer position $\mathbf{x}_{n_{s,r}}$ out of the model space using the focusing equations implies a transmission of the positioning effect into the slope $p_{s,n_{s,r}}$ sensitivity with respect to m. The link is established while zeroing the derivative of the augmented functional with respect to $\mathbf{x}_{n_{s,r}}$:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}_{n_{s,r}}} = \bar{\mathbf{u}}_1 \frac{\partial T_{s,r,n_{s,r}}}{\partial \mathbf{x}_{n_{s,r}}} + \Delta p_{s,n_{s,r}} \frac{\partial p_{s,n_{s,r}}}{\partial \mathbf{x}_{n_{s,r}}} + \bar{\mathbf{u}}_2 \frac{\partial p_{r,n_{s,r}}}{\partial \mathbf{x}_{n_{s,r}}}.$$

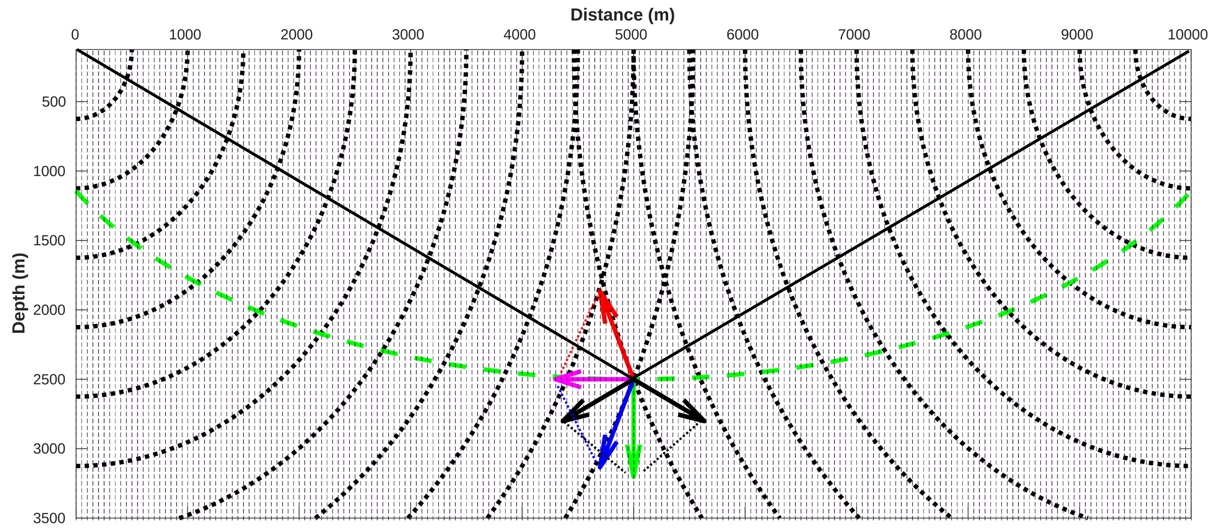

Figure 3: Three terms of $\partial\mathcal{L}/\partial\mathbf{x}_{n_{s,r}}$ with related colors. Added magenta arrow is the sum of blue and red arrows.

IV - Synthetic and real data application

•**Marmousi case :** Tomography setup → 6708 scattering events, streamer acquisition, multi-scale approach. FWI setup → fixed-spread acquisition, frequencies $[4, 6, 8, 10, 12, 14$ and 16 Hz$]$.

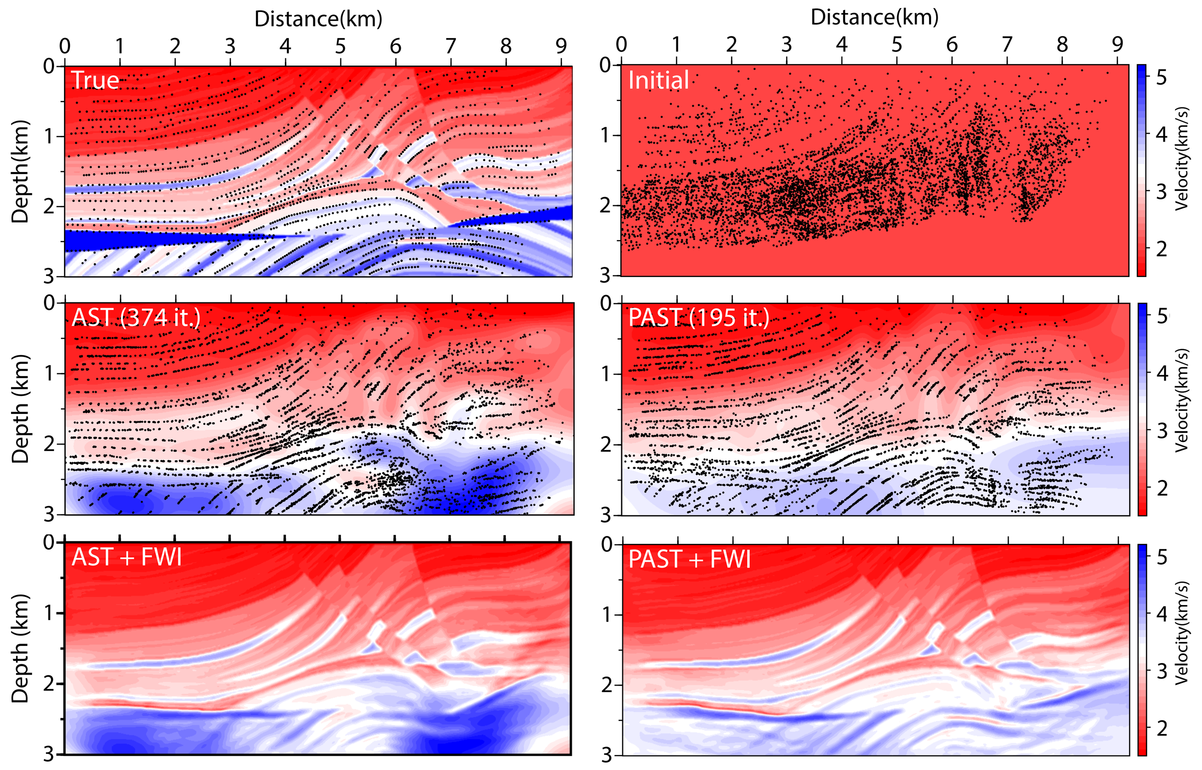

Figure 4: AST and PAST inversion results and their FWI.

✓ Good velocity reconstruction in the reservoir.
✓ Improved convergence with respect to AST.

•**Real data application :** Broadband streamer acquisition, 50000 scattering events, multi-scale approach, passive anisotropy (TTI) parameters.

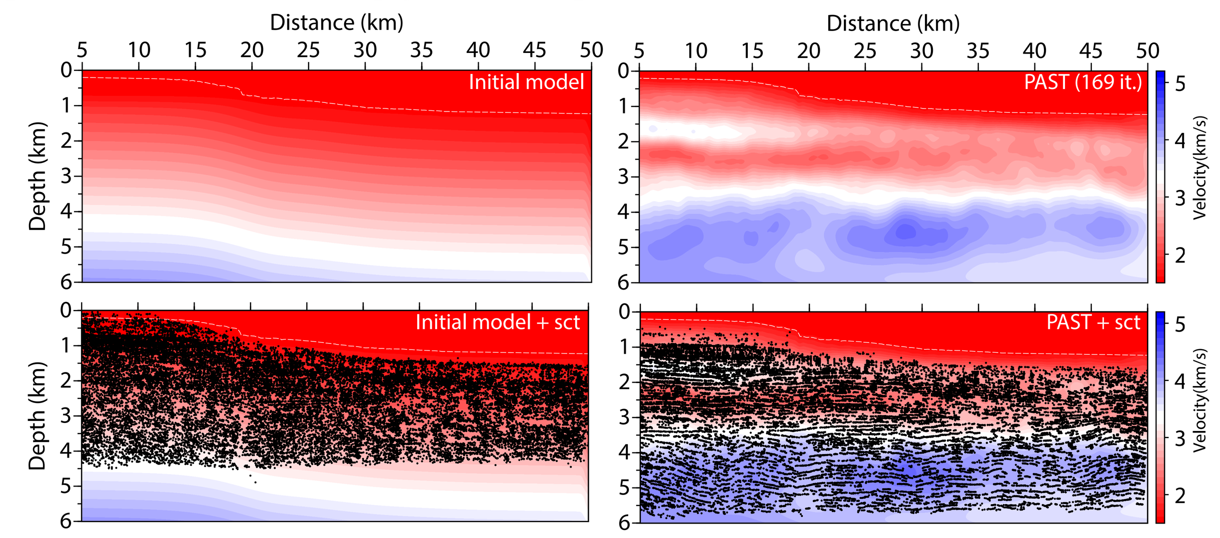

Figure 5: PAST inversion results after 169 iterations.

✓ Velocity model validated with well logs.

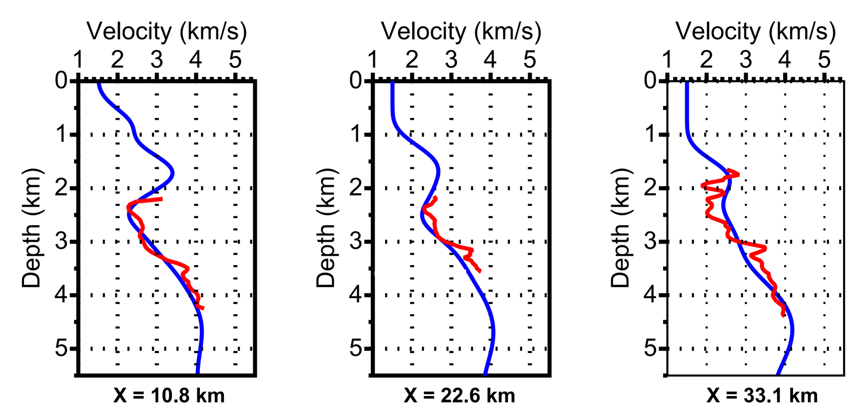

Figure 6: Comparative logs with respect to well data (red).

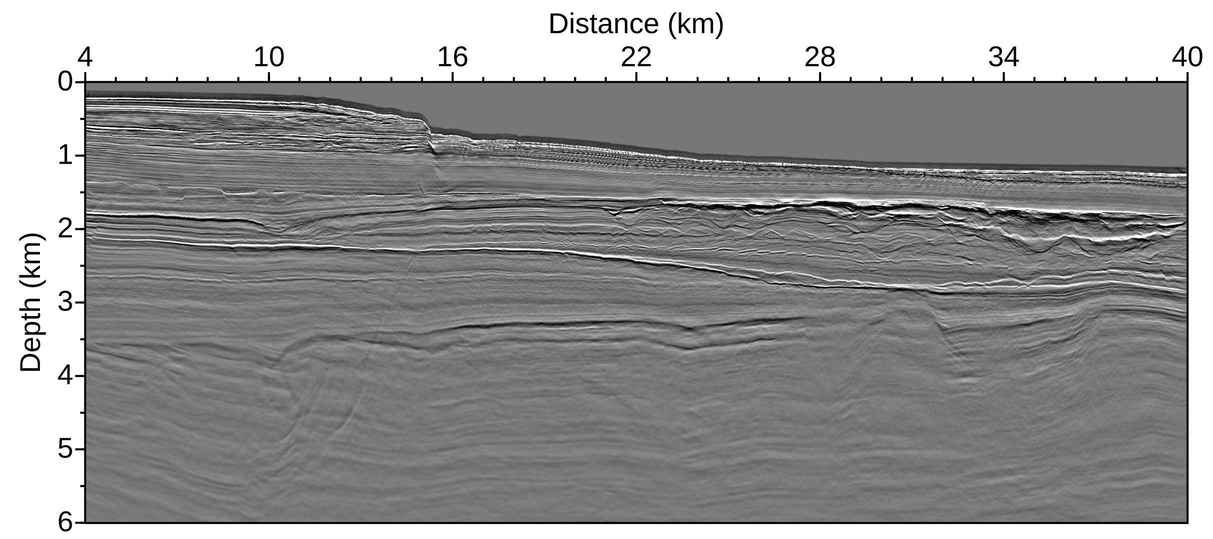

Figure 7: Image obtained through TTI Kirchhoff migration.

✓ Well focused image and coherent result with respect to previous studies.
✓ Flat events in the Common Image Gathers.

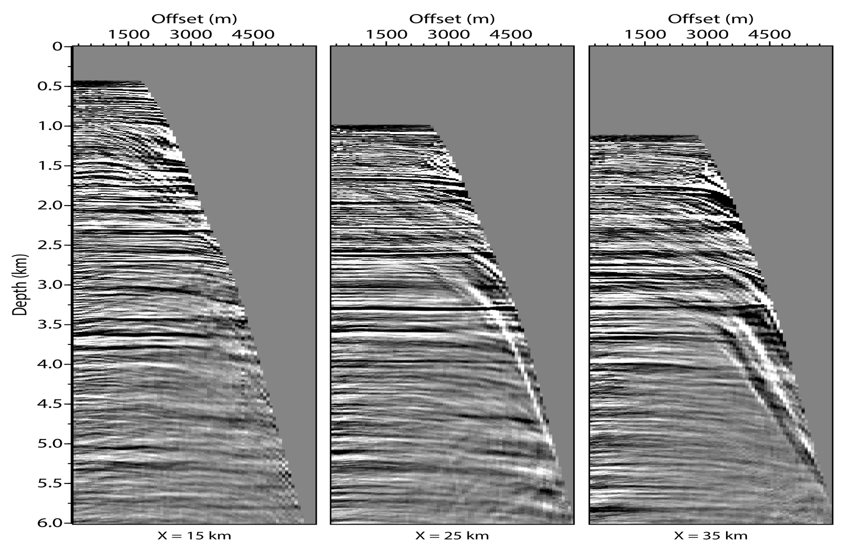

Figure 8: Common Image Gathers (CIG)

Conclusion

We present a strategy to tackle the velocity-position coupling in the context of slope tomography. An induced consistency between the scatterers position and the background velocity field is achieved through a variational projection approach. We benchmark our method and validate it on a real data case. The results exhibit an improvement under this formulation with respect to a joint inversion.
A similar approach could be employed in other contexts like the hypocenter-velocity problem.

References

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