An Evolutive Linear Kinematic Source Inversion

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November 22, 2022

Abstract

Accurate kinematic models are fundamental to enhance our knowledge of the seismic cycle as well as to improve surface ground motion prediction. However, the solution of the ill-posed kinematic inverse problem is non-unique (e.g., Cohee & Beroza, 1994; Wald & Heaton, 1994; Cotton & Campillo, 1995 and Minson et al., 2013) and, according to current acquisition systems surrounding active faults, this problem is highly underdetermined, in spite of its rather simple formulation as a linear inverse problem. Non-linear formulations of the problem, based on model reduction strategies, alleviate the underdetermination of the problem. However, non-linear formulations imply drastic assumptions on the rupture history and they complicate the use of linear algebra tools to assess the uncertainties of results. Regardless of the assumed inverse formulation, the incorporation of physical constrains and prior information into the inverse problem is necessary to provide more robust and plausible solutions. In this work (Sanchez-Reyes et al. 2018), we present a new hierarchical linear time domain kinematic source inversion method able to assimilate data traces through evolutive time windows. This progressive approach, both on the data and model spaces, does require mild assumptions based on prior knowledge or preconditioning strategies on the slip rate local gradient estimations. Contrary to similar approaches (Fan et al., 2014), this strategy benefits from the sparsity and causality of the seismic rupture while still ensuring the positivity of the solution. While standard regularization terms are used for stabilizing the inversion, strategies based on parameter reduction leading to a non-linear relationship between the source history and the observed seismograms are avoided. Rise time, rupture velocity and other attributes can be extracted later on from the slip-rate inversion we perform. . Satisfactory results are obtained on synthetic benchmarks proposed by the Source Inversion Validation project (Mai et al. 2016) and for the 2016 M$_w$7.0 Kumamoto mainshock. Our specific formulation combined with simple prior information, as well as numerical results obtained so far, yields interesting perspectives for a quasi-real-time implementation and to ease the uncertainty quantification of such ill-conditioned problem.

An evolutive linear kinematic source inversion

Inverting while recording: reducing time-space ambiguity

Sanchez-Reyes H. S.[1]
with Brossier R.[1], Cruz-Atienza V. M.[4], Métivier L.[1,3], Tago J.[2] and Virieux J.[1]

13th December 2018, 9:15 - 9:30

1. Institut des Sciences de la Terre, UGA, France
2. Facultad de Ingeniería, UNAM, Mexico
3. Laboratoire Jean Kuntzmann, UGA, France
4. Instituto de Geofísica, UNAM, Mexico

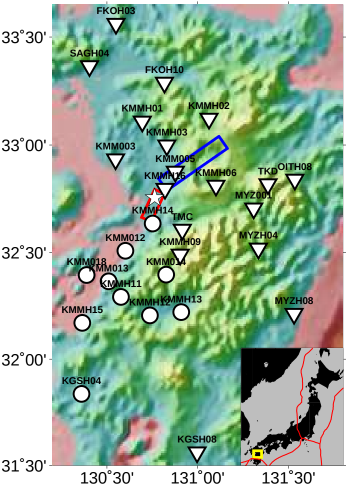
2018 AGU Fall Meeting

Thanks AGU and authors for sharing your presentations with visually impaired attendees

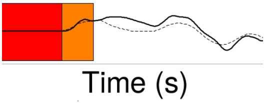
It is hard to follow the meeting when you can not see what is going on the screen!

In the next 10-12 minutes:

2016 M_W7.0 Kumamoto earthquake



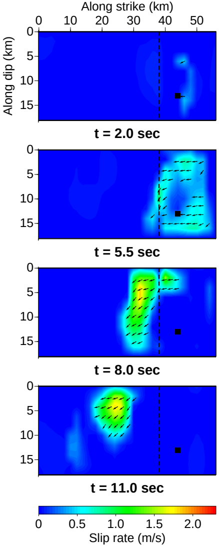
Inverting progressively increasing data time-windows



While still:

- respecting positivity
- enforcing causality
- including possible prior information

Inverted source history



Outline

Motivation

Methodology description (2016 M_w7.0 Kumamoto earthquake)

Conclusions and perspectives

Outline

Motivation

Motivation: time-space ambiguity

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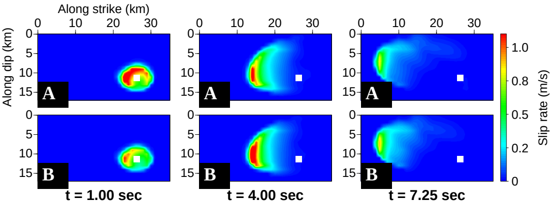
Conclusions and perspectives

Imagine two different rupture histories (**A** and **B**)!



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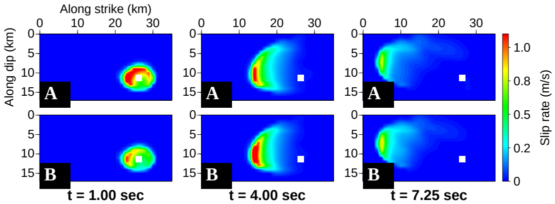
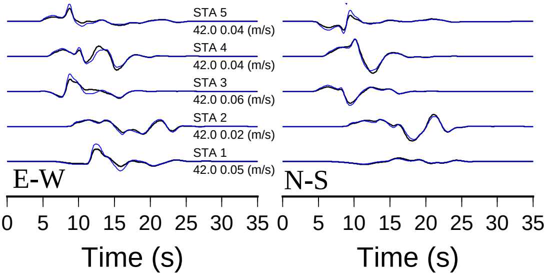


Which data sets are slightly different ($\leq$ 1Hz):

Rupture A vs Rupture B



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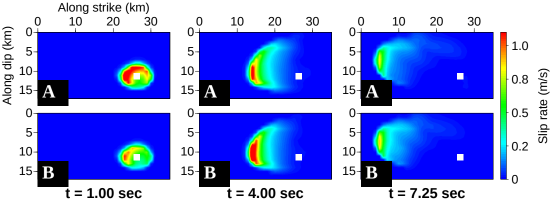

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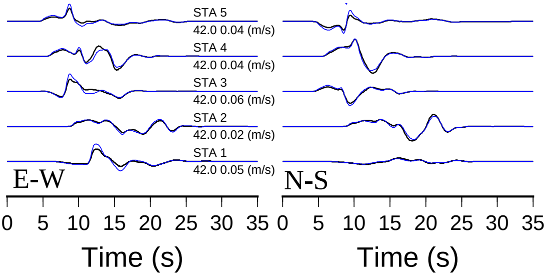
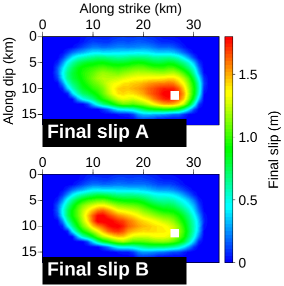


Different final slip distributions



Motivation:

Develop an inverse method able to reconstruct the correct source history by assimilating and inverting the data in a different way.

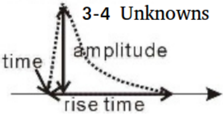
JGR: Solid Earth 10.1029/2017JB015388

Motivation

Methodology description (2016 M_w7.0 Kumamoto earthquake)

Conclusions and perspectives

Non-linear



Few parameters per node:

- starting time
- duration
- max amp
- angle

Advantage:

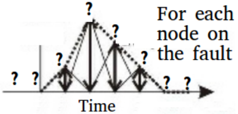
Explicit physical parametrization

Disadvantage:

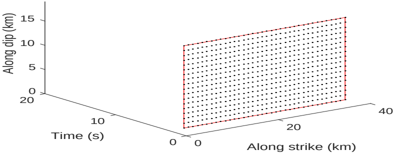
Strong assumptions impacting results

With drawbacks when assessing uncertainties

Linear



Search for time-space history (3D)



Disadvantage:

Large number of unknowns (tens of thousands)

Advantage:

Good for uncertainty assessment

☞ 1. Linear forward modeling:

$$\underbrace{\underbrace{\underline{d}}_{\textit{seismograms}} = \underbrace{\underline{\underline{G}}}_{\text{wave propagator}} * \underbrace{\underline{m}}_{\text{rupture model}}}_{\text{Linear relation}}$$

Ingredients of this linear time domain formulation

☞ 1. Linear forward modeling:

$$\underbrace{\underbrace{\underline{d}}_{\textit{seismograms}} = \underbrace{\underline{\underline{G}}}_{\text{wave propagator}} * \underbrace{\underline{m}}_{\text{rupture model}}}_{\text{Linear relation}}$$

☞ 2. L2 Norm misfit function:

$$\mathcal{C}(\underline{m}) = \text{Data misfit} + \overbrace{\begin{matrix}\text{Model misfit}\\ \text{based on prior info}\\ \text{and rupture physics}\end{matrix}}^{\textbf{Very important}}$$

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3. Newton equation using data gradient & model gradient: $\underline{\gamma} = \underline{\gamma}_{data} + \underline{\gamma}_{model}$

$$\underbrace{\underline{\underline{H}}}_{\text{Hessian}} \underline{\Delta m} = - \underbrace{\underline{\gamma}}_{\textit{Gradient}}$$

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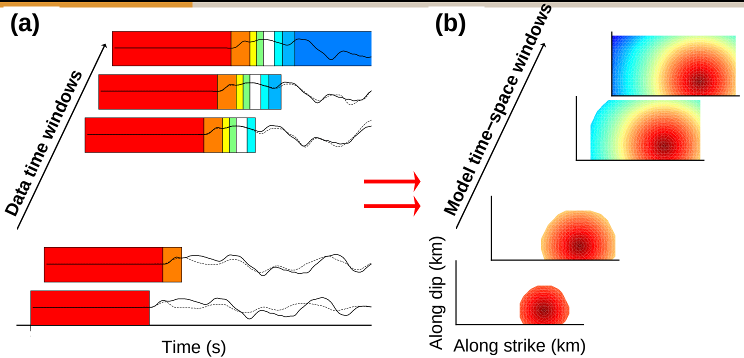
3. **Newton equation using data gradient & model gradient:** $\underline{\gamma} = \underline{\gamma}_{data} + \underline{\gamma}_{model}$

same kernel!

$$\underbrace{\underline{\underline{H}}}_{\text{Hessian}} \; \underline{\Delta m} = - \underbrace{\underline{\gamma}}_{Gradient} \longrightarrow \underline{\gamma}_{data} = \underbrace{\underline{\underline{G}}}_{\text{wave propagator}} * \text{Data residuals}$$



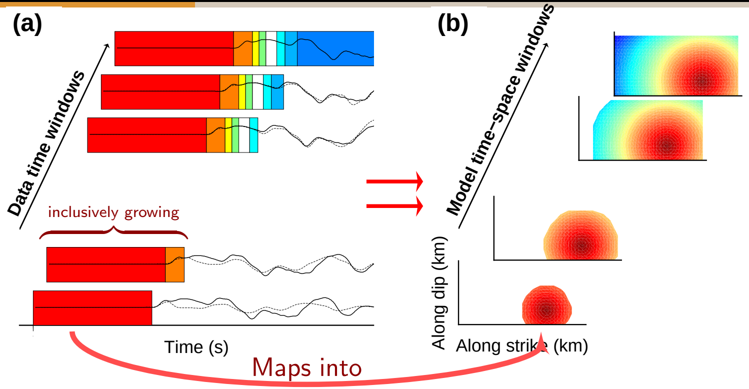
Assumptions:

- Previous calibration of data and model time-space windows.
- Requires a synthetic rupture for the calibration.
- Pre-computed Green functions.

Benefits:

- Rough prediction of wave packets to come!
- Only residuals need to be explained!
- Residuals map mostly into the new allowed rupture zone.

Data assimilation and time-space model expansion



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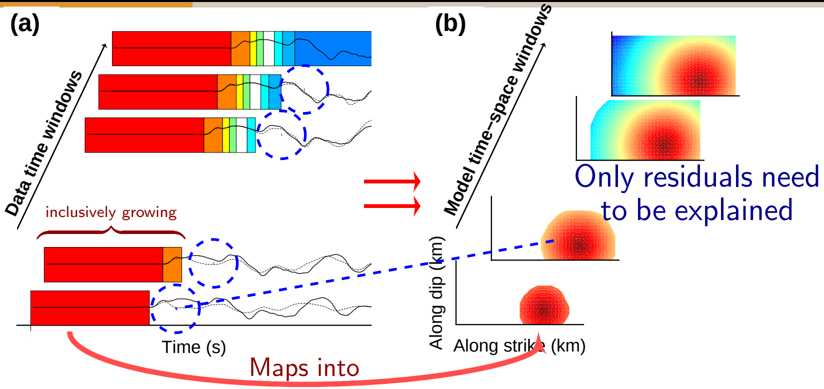

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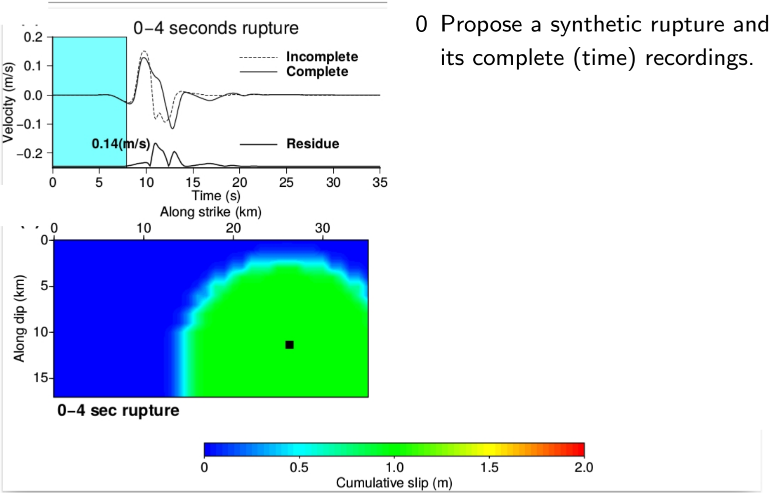
Given a source/receiver geometry it is possible to define our data time windows and time-space model growth using a simple synthetic rupture.



0 Propose a synthetic rupture and its complete (time) recordings.

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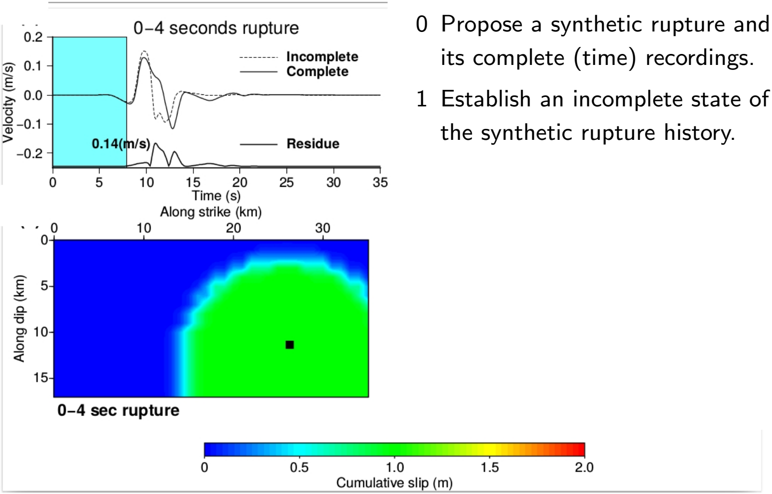
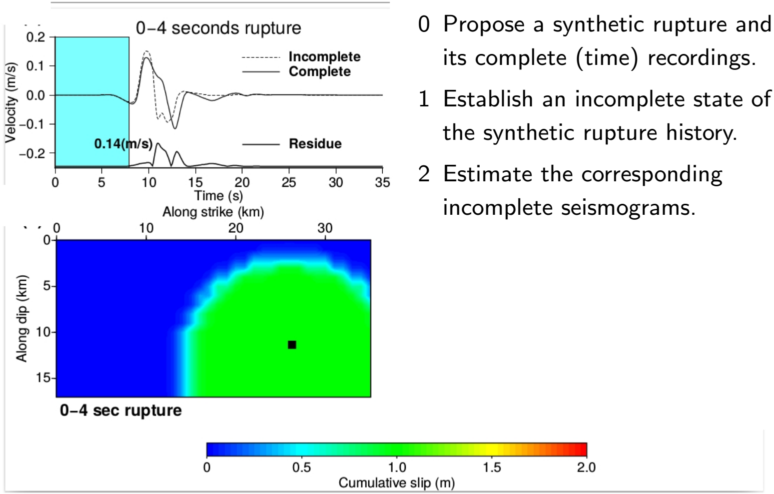


0 Propose a synthetic rupture and its complete (time) recordings.

1 Establish an incomplete state of the synthetic rupture history.

Time windowing pre-calibration is required

Given a source/receiver geometry it is possible to define our data time windows and time-space model growth using a simple synthetic rupture.



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2 Estimate the corresponding incomplete seismograms.

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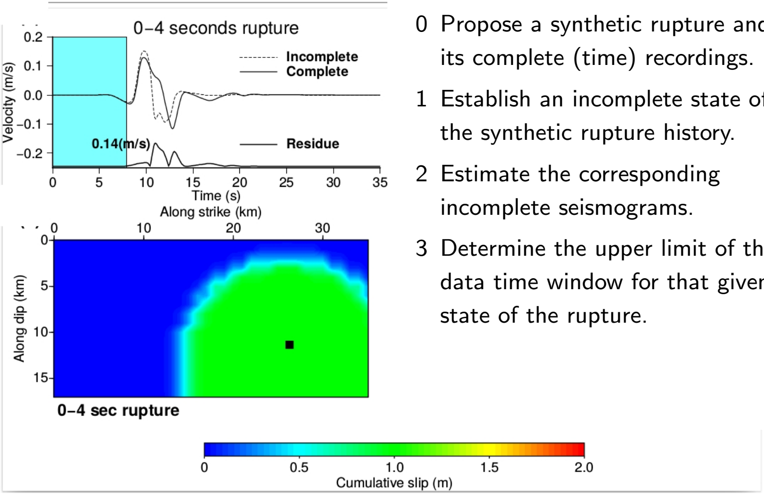

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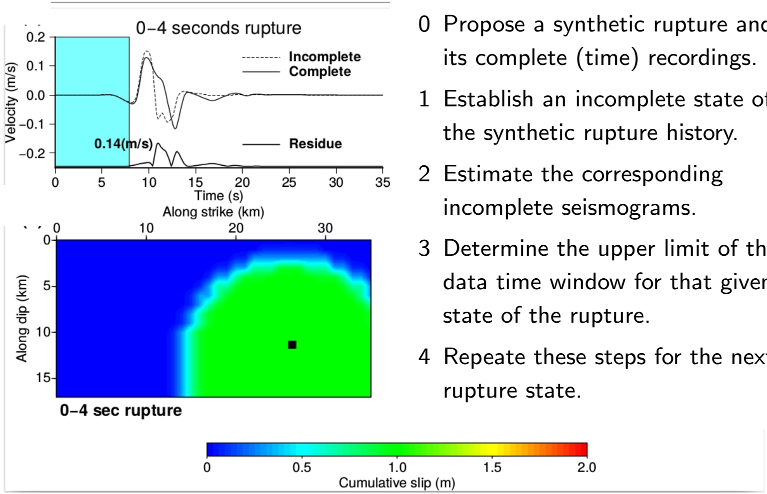


0 Propose a synthetic rupture and its complete (time) recordings.

1 Establish an incomplete state of the synthetic rupture history.

2 Estimate the corresponding incomplete seismograms.

3 Determine the upper limit of the data time window for that given state of the rupture.

4 Repeate these steps for the next rupture state.

Rupture physics are not yet included!

$$\mathcal{C}(\underline{m}) = \text{Data misfit}_{|_{t_0}^{t_1}}$$

Model regularization and data gradient preconditioning

Model regularization and gradient preconditioning ARE REQUIRED.

Data driven model preconditioning:

- Depth preconditioning to mitigate surface acquisition footprint.
- Gradient smoothing to enforce spatial coherence.

Model regularization:

- Upper and lower bounds of rupture velocity.
- Expected zones of minimum slip (fault edges).
- Min and Max slip rate bounds.
- Other prior information (rake angle).

$$\mathcal{C}(\underline{m}) \quad = \quad \text{Data misfit} \quad + \quad \overbrace{\begin{array}{c}\text{Model misfit}\\ \text{based on prior info}\\ \text{and rupture physics}\end{array}}^{\textbf{Very very important}}$$

Preconditioning and regularization can also evolve during the inversion!

Inversion results from previous data windows can be used to enhance our prior information.

This strategy helps to reduce the footprint of the regularization from the final results.

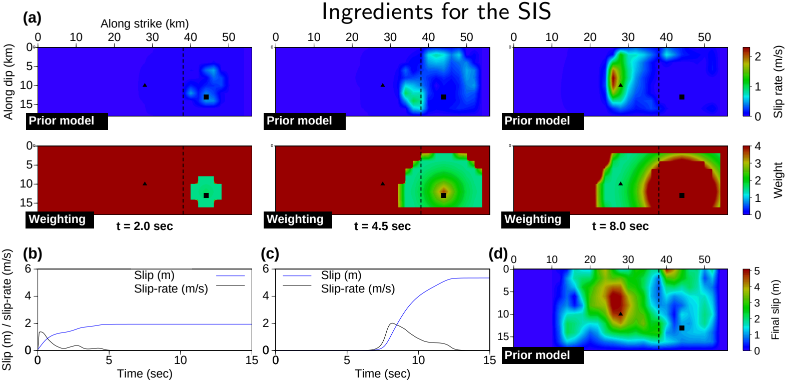
Standard Inversion Strategy (SIS): **(Traditional approach)**

- The full recordings are inverted.
- During the inversion, **NO EVOLUTION** of:

  ⟶ The prior model (reconstructed from Asano and Iwata (2016))
  (other prior model or information can be injected)

  ⟶ and its weigthing (defined based on physics and after several tests).

Ingredients for the SIS



Progressive Inversion Strategy (PIS): (**New approach**)
inspired by Kikuchi and Kanamori (1982)

- Progressively increasing data time windows are inverted.
- The prior model and its weigthing **EVOLVE** during the inversion.

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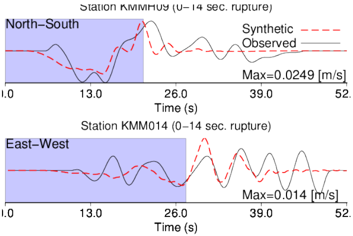
Data inverted – Data predicted



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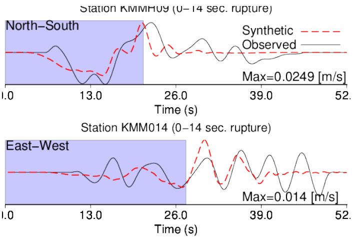
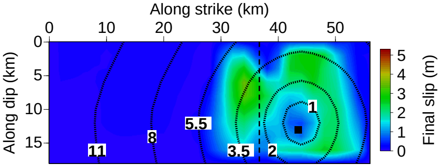
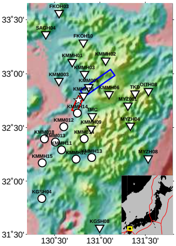


Cummulative slip after 6 seconds

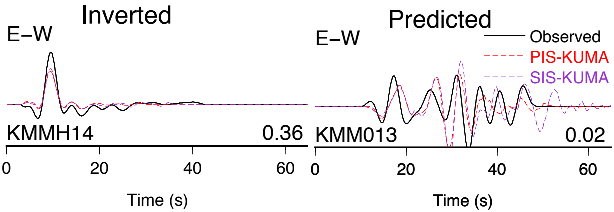


Such changes in the regularization help to reduce its footprint in our results.

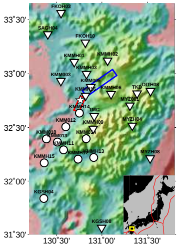


▽ 57 Recordings inverted

○ 33 Recordings predicted



Standard full-time inversion (SIS) VS Progressive Inversion (PIS)



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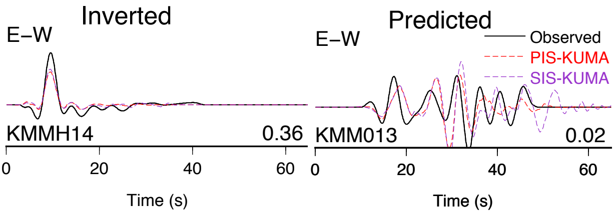

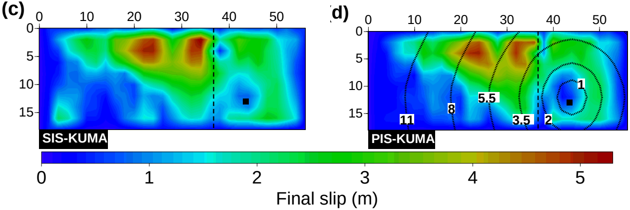

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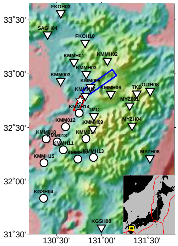

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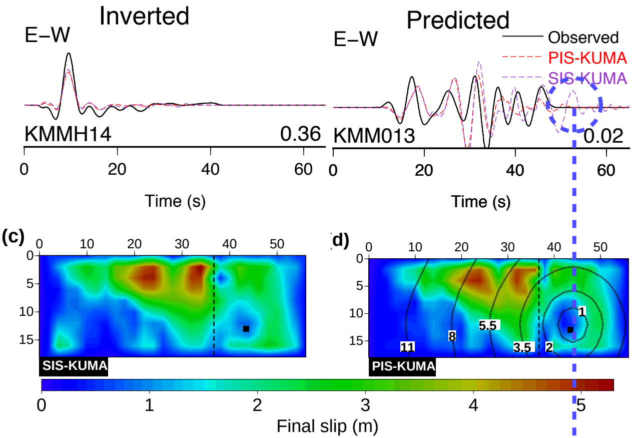
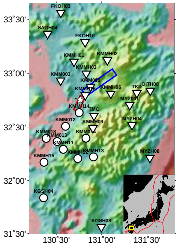


PIS predicts better



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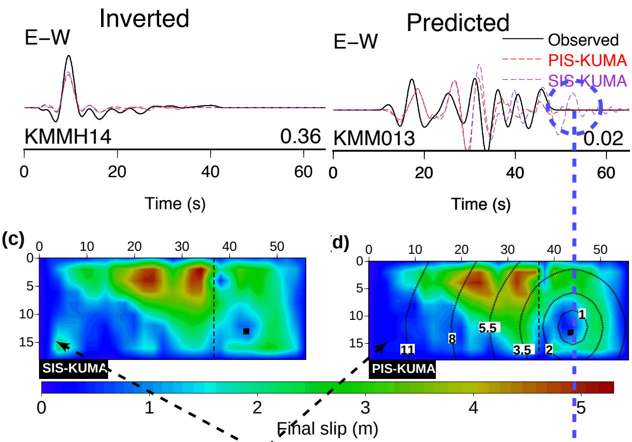


With the same misfit, SIS and PIS lead to different solutions!

PIS predicts better

Outline

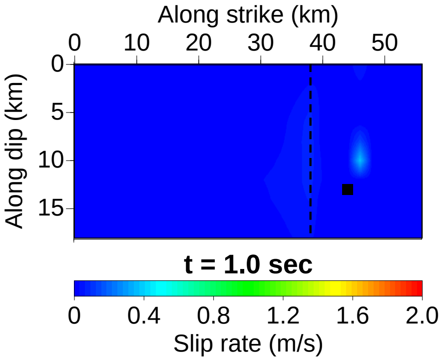
Motivation

Methodology description (2016 M_w7.0 Kumamoto earthquake)

Conclusions and perspectives

Some important conclusions:

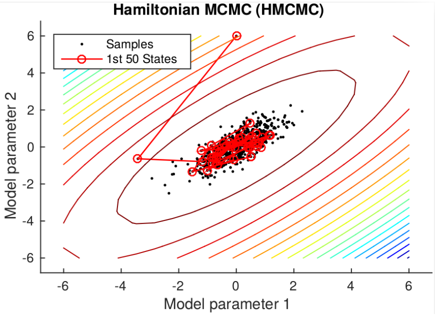
- ☞ Preserving the linearity of the forward problem:
  physics are enforced through model preconditioning/regularization rather than applying model-reduction strategies.
- ☞ Progressive inversion strategy reduces space/time leakage by honoring causality
- ☞ Uncertainty quantification easier with linear forward problem.



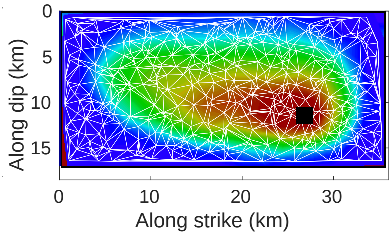
For assessing uncertainties:

- Hamiltonian MCMC (HMCMC): Possible, efficient and attractive.
- Sequential MCMC: Possible and able to handle data-assimilation.
- Reverse Jump HMCMC (RJHMCMC): Possible and very attractive.

How certain are our results?



According to the data, what is the best mesh to use?



Advert: If you want to read more about this work

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JGR

Journal of Geophysical Research: Solid Earth

RESEARCH ARTICLE

10.1029/2017JB015388

Key Points:

- An alternative linear inverse formulation for kinematic source reconstruction is presented
- Such formulation can invert progressively growing data time windows while spanning the model space
- Promising advantages of this method are found, thanks to the preservation of causality and sparsity

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http://hugosanrocks.github.io/

Thanks for listening!

Immigrants are not "bad hombres"!!

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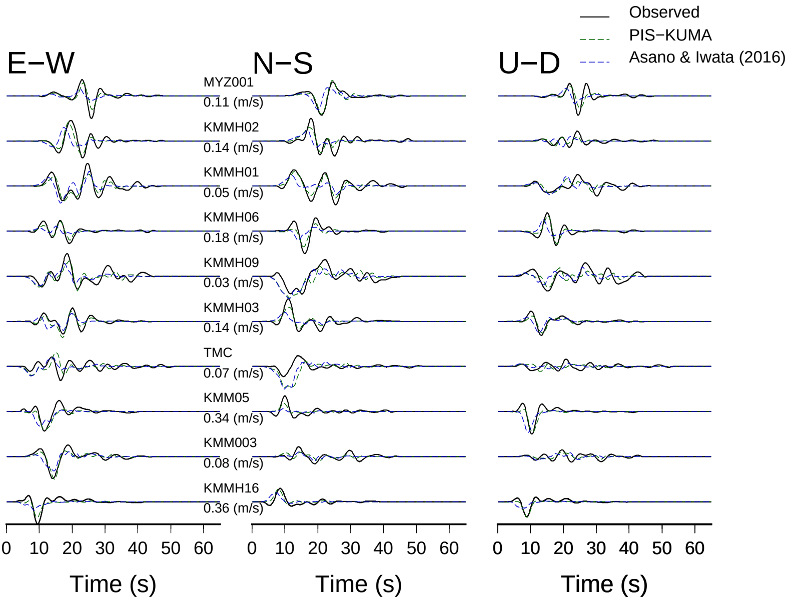
Sánchez-Reyes, H., Tago, J., Métivier, L., Cruz-Atienza, V., and Virieux, J. (2018). An evolutive linear kinematic source inversion. *Journal of Geophysical Research: Solid Earth*, 123.

Prior model (Asano & Iwata, 2016) misfit vs PIS results



PIS Workflow: How it works?