

Numerical Models for the DRESHDYN Precession Dynamo Experiment

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Abstract

More than 100 years ago, Henri Poincaré in his pioneering study showed that the inviscid base flow in a precessing spheroid is described by a constant vorticity solution, the spin-over mode. Since then there have been repeated discussions whether the geodynamo is driven (or at least influenced) by precession. More recently, precession has also been considered as an important mechanism for the explanation of the ancient lunar dynamo. Experiments with precessing fluids in cylindrical and in spherical geometry showed that precession indeed is an efficient mechanism to drive substantial flows even on the laboratory scale without making use of propellers or pumps. A precession dynamo experiment is currently under construction within the project DRESHDYN (DREsden Sodium facility for DYNamo and thermohydraulic studies) at Helmholtz-Zentrum Dresden-Rossendorf (HZDR) in which a precession driven flow of liquid sodium will be used to drive dynamo action. In the present study we address related numerical and experimental examinations in order to identify parameter regions where the onset of magnetic field excitation will be possible. Preliminary kinematic dynamo models using a prescribed flow field from hydrodynamic simulations, exhibit magnetic field excitation at critical magnetic Reynolds numbers around $R_{mc} \approx 430$, which is well within the range of the planned liquid sodium experiment. Our results show that large scale inertial modes excited by precession are able to excite dynamo action when their structure is sufficiently complex, i.e. the forcing is sufficiently strong. More advanced models that take into account the container's finite conductivity show that boundary conditions may play an important role, but the critical magnetic Reynolds number will still be achievable in the planned experiment. Finally, we discuss the role of turbulent flow fluctuations for the occurrence of dynamo action.

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The precession dynamo experiment at HZDR

Motivated by the idea of a precession-driven flow as energy source for the early geodynamo or the ancient lunar magnetic field, a large precession dynamo experiment is under development at HZDR.

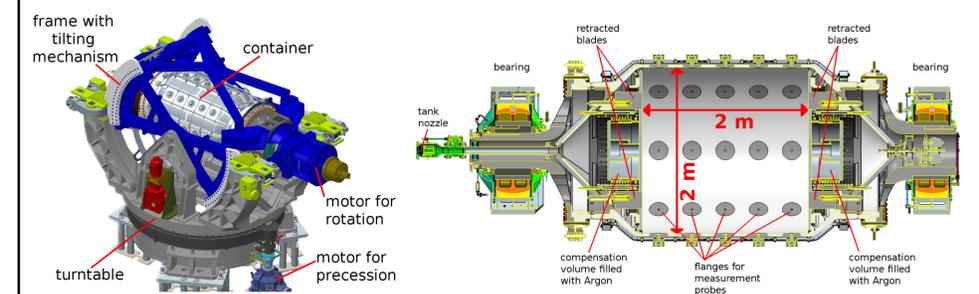


Fig. 1: Sketch of the facility (left) and the container (right). The cylinder has radius $R=1$ m and height $H=2$ m. The tilt between rotation axis and precession axis can be varied from 45° to 90° . The cylinder will be filled with liquid sodium with $T \approx 120^\circ$ C.

rotation rate	precession rate	nutation angle	Reynolds	magnetic Reynolds	aspect ratio	precession ratio
$f_c = \frac{\Omega_c}{2\pi}$	$f_p = \frac{\Omega_p}{2\pi}$	α	$Re = \frac{\Omega_c R^2}{\nu}$	$Rm = \frac{\Omega_c R^2}{\eta}$	$\Gamma = \frac{H}{R}$	$Po = \frac{\Omega_p}{\Omega_c}$
0 ... 10 Hz	0 ... 1 Hz	$45^\circ \dots 90^\circ$	up to 10^8	up to 700	2	0 ... 0.1

- natural forcing allows efficient flow driving without propellers or pumps
- Gans' experiments in the 70's yield field amplification by a factor of 3
- precession dynamos are found in simulations around $Rm \sim O(10^3)$

We conduct numerical simulations with SEMTEX (Spectral Element-Fourier method) and flow measurements in a down-scaled water experiment with Ultrasonic Doppler Velocimetry (UDV) (Fig. 2)

$$\frac{\partial}{\partial t} \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla P - 2\Omega_p \times \mathbf{u} + \nu \nabla^2 \mathbf{u} \text{ and } \mathbf{u}_{bc} = \Omega_c \times \mathbf{r}$$

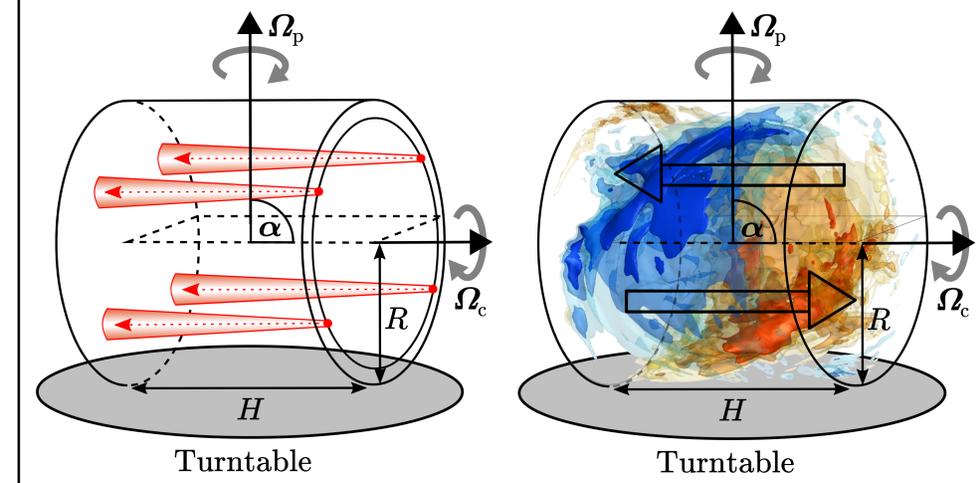


Fig. 2: Left: set-up of the water experiment with four UDV probes mounted at one end cap of the cylinder. The probes provide instantaneous profiles of the velocity component in direction of the ultrasound beam. Right: Iso-surfaces of the axial velocity from simulations at $Re=10^4$ and $Po=0.10$. The arrows illustrate the overall recirculation flow. The nutation angle is fixed at $\alpha = 90^\circ$.

Inertial waves & amplitudes

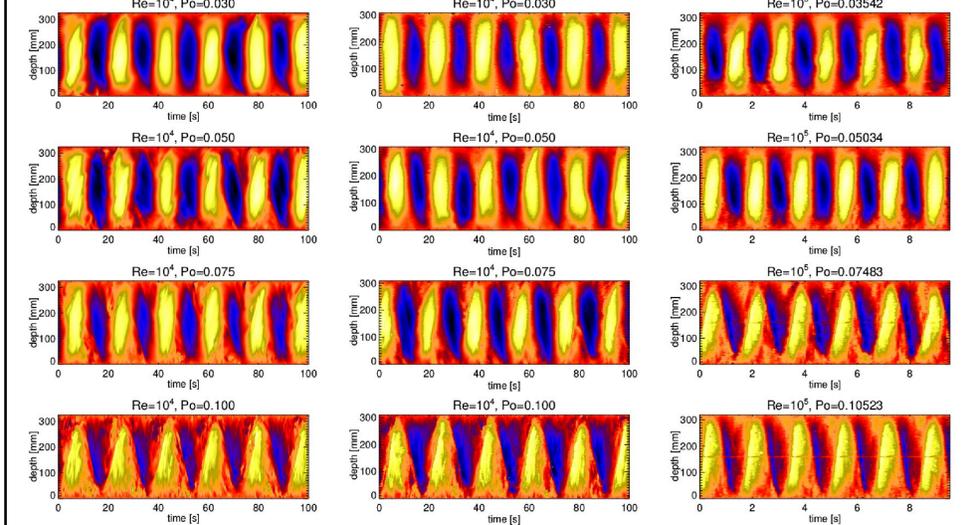


Fig. 3: Axial profiles of u_z for $Re=10^4$ (left and center) and $Re=10^5$ (right). From top to bottom: $Po = 0.03, 0.05, 0.075, 0.10$. The left column shows results from numerical simulations and the central and the right column show results from experimental measurements.

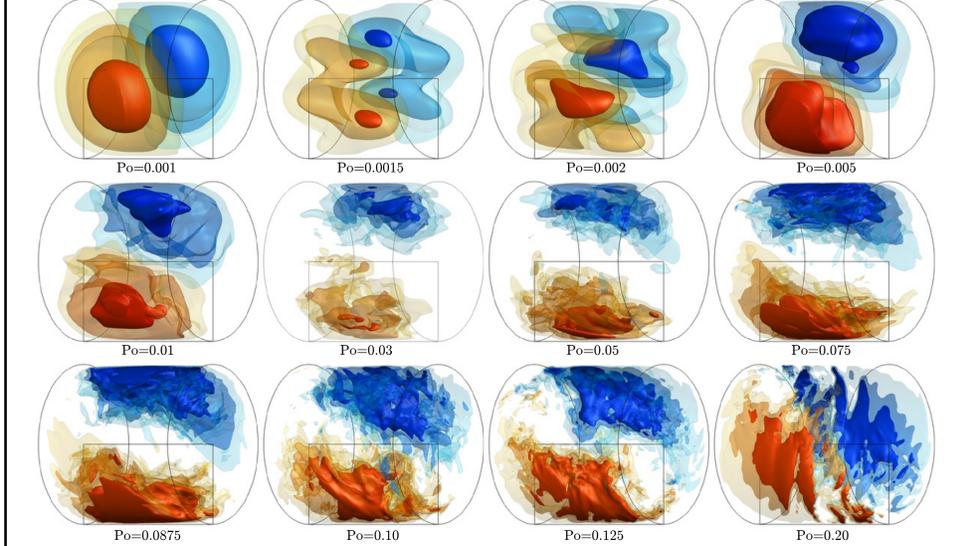


Fig. 4: Three-dimensional structure of the axial velocity for increasing Po . The behavior of the flow varies from a stationary state (top left) to the occurrence of free azimuthal drifting Kelvin modes and turbulent fluctuations on top of a mean flow (bottom row).

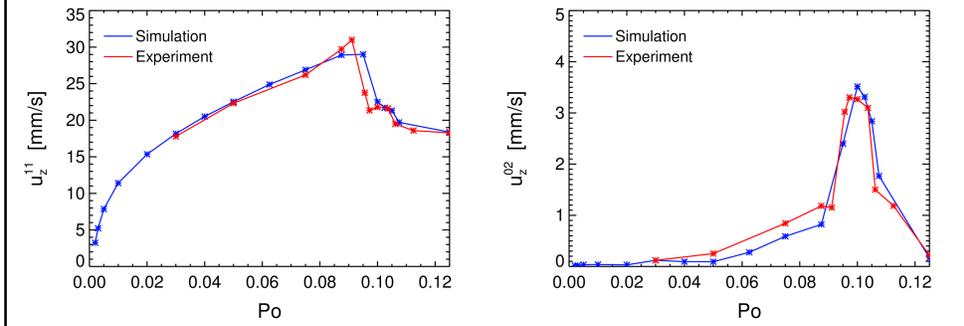


Fig. 5: Amplitude of forced mode $(m,k)=(1,1)$ (left) and of the axisymmetric mode $(m,k)=(0,2)$ (right).

- laminar flow consisting of large scale inertial modes (standing waves in precession frame) with only weak time-dependent contributions

Kinematic dynamos

The time-averaged velocity fields obtained from hydrodynamic simulations constitute the basis for kinematic dynamo models. The magnetic induction equation is solved numerically using pseudo-vacuum boundary conditions.

$$\frac{\partial}{\partial t} \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B})$$

Fig. 6: Left: Growth rates versus Rm for various (time-averaged) velocity fields obtained at different Po . Center: Growth rates for combinations of azimuthal modes ($m=0, m=1, m=2$) for the flow obtained at $Po=0.1$. Right: Growth rates for the full flow at $Po=0.1$ with an external layer mimicking the finite conductivity of the container walls.

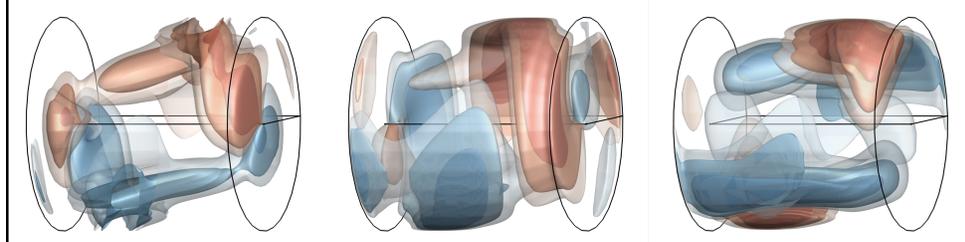


Fig. 7: Structure of the magnetic field at 12.5, 25, 50% of its maximum value. From left to right: B_r, B_φ, B_z . The field structure propagates around the cylinder axis. $Re=10^4$ and $Po = 0.1$.

- dynamo action only for flow fields above $Po = 0.095$ with minimum $Rm^{crit} = 430$ in case of flow at $Re=10^4$ and $Po = 0.1$.
- combination of axisymmetric flow ($m=0$) and directly forced mode ($m=1$) required
- outer layer with finite electrical conductivity converges to idealized pseudo-vacuum BC

Dynamos possible at parameters that will be achievable in the planned experiment but only in a narrow regime with $m = 0$ mode

Scaling to sodium device

- $m = 0$ mode emerges at smaller Po when Re is increased, and UDV measurements indicate asymptotic behavior with $Po^c \approx 0.06 \dots 0.07$ for $Re \geq 10^5$
- consideration of temporal fluctuations can be done by mean-field approach

$$\mathcal{E} = \langle \mathbf{u} \times \mathbf{b} \rangle = \alpha \langle \mathbf{B} \rangle + \beta \nabla \times \langle \mathbf{B} \rangle$$

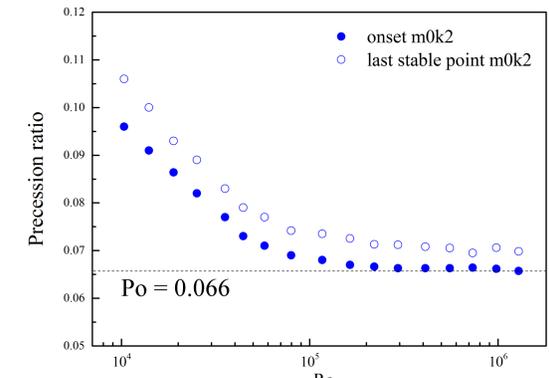


Fig. 8: Regime with occurrence of the axisymmetric mode from UDV measurements with increasing Re