# Modeling the Remote Sensing Reflectance of Highly Turbid Waters

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#### Abstract

Several models relating remote sensing reflectance to water inherent optical properties (IOPs) have been developed. In particular, the reflectance is expressed as a function of a parameter u which is related to the absorption and backscattering coefficients. We note that the quadratic model reported by Gordon et al (1988) has been widely accepted and validated. A more recent model by Lee et al (2004) separated the contributions by water and particle scattering. Most models however, only consider oceanic waters where scattering is low. This is not the case in coastal or inland waters with high suspended sediment load. Using HydroLight simulations in waters with high scattering coefficient values, we found that the quadratic relation is not sufficient to describe the corresponding remote sensing reflectance. A polynomial of at least fourth degree is required to fit the simulation results at high u. Monte Carlo simulations were conducted to investigate the relation between the remote sensing reflectance and the u parameter at similar IOP values. Results of Monte Carlo simulations confirm the quartic relation derived from HydroLight. Application of this derived relationship in relating IOPs to remote sensing reflectance will avoid significant errors in waters of high turbidity.

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#### 1. Introduction

In ocean colour remote sensing, subsurface remote-sensing reflectance  $(r_{rs})$  of water can be linked to its inherent optical properties (IOPs) by various models. The use of such models allow for quick calculations of IOPs from  $r_{rs}$  and vice versa, eliminating the need to solve the radiative transfer equation.

In particular,  $r_{rs}$  is expressed as a function of a parameter u which is related to the absorption and backscattering coefficients. We note that the quadratic model by Gordon *et al* (5) has been widely accepted and validated, at least for case 1 waters. A more recent model by Lee *et al* (7) separated the contributions from water and particle scattering. Most models however, only consider oceanic waters where scattering is low. The relationship between  $r_{rs}$  and u may deviate from these models in coastal or inland waters with high suspended sediment load.

By running HydroLight (3,4) simulations in waters with high scattering coefficient values, we found that neither of the models (5,7) were sufficient to describe the corresponding  $r_{rs}$  at high u. A polynomial of *at least* fourth order was required. The non-linear increase may be attributed to multiple scattering by suspended particles in highly turbid waters (15). Monte Carlo simulations (6) were conducted with similar water types and compared to the results from HydroLight. The Monte Carlo results clearly validated the trend observed from HydroLight simulations.

By incorporating the water-particle separation by Lee *et al* (7), we derived a general model relating  $r_{rs}$  and IOPs for waters of any turbidity, excluding correction for trans-spectral effects (9). The new model is similar to that of Lee *et al* (7) at low *u*, but also accounts for  $r_{rs}$  at high *u*. Application of this derived relationship in relating IOPs to  $r_{rs}$  will avoid significant errors in waters of high turbidity.

#### 2. Background

The model by Gordon et al (5) is

$$r_{rs} = l_1 u + l_2 u^2, (1)$$

where  $l_1$  and  $l_2$  are model parameters, and u is defined as

$$u = \frac{b_b}{a + b_b},\tag{2}$$

where a is the total absorption coefficient, and  $b_b$  is the total backscattering coefficient of the water.

Lee *et al* (7) found that it was possible for waters with different a and  $b_b$  to have the same u due to water and particle scattering contributions to  $b_b$ . They proposed the following model accounting for separate contributions from water and particles

$$r_{rs} = g_w u_w + G_0 u_p \left[ 1 - G_1 \exp(-G_2 u_p) \right], \tag{3}$$

where  $g_w$ ,  $G_0$ ,  $G_1$ ,  $G_2$  are model parameters, and

$$u_w = \frac{b_{bw}}{a + b_b},\tag{4}$$

$$u_p = \frac{b_{bp}}{a + b_b}.$$
(5)

The backscattering coefficients  $b_{bw}$  and  $b_{bp}$  are that of water molecules and suspended particles respectively. It should be noted that the model parameters vary significantly with solar and sensor geometry (2,13,14). For the purpose of comparison, we will consider the case of a nadir viewing sensor in this paper. The two models (Eq. 1 and Eq. 3) will be referred to as Gordon88 and Lee04 respectively.

#### 3. Approach

First, we note that variable u lies within the limits 0 < u < 1 for any type of water. It is thus possible to fully simulate water types over the complete range of u.

#### a. HydroLight Simulations

We used the HydroLight code (10,12) to compute  $r_{rs}$  from a wide range of CDOM absorption and particle backscattering coefficients, such that the complete range of u is simulated. The input values used in HydroLight are recorded in Table 1.

The spectral relationships (1,8) used to compute the CDOM absorption coefficient  $a_g(\lambda)$  and particle backscattering coefficient  $b_{bp}(\lambda)$  are

$$a_g(\lambda) = a_g(440) \exp[-S(\lambda - 440)]$$
 (6)

$$b_{bp}(\lambda) = b_{bp}(550) \left(\frac{550}{\lambda}\right)^{\gamma} \tag{7}$$

The particle scattering phase function was chosen to be a Fournier-Forand phase function (4) with  $b_{bp}/b_b = 0.018$ .

The choice of water component types and wavelengths are of little consequence for the simulations, as the HydroLight code only uses the resulting absorption and backscattering coefficients for its computation.

From the HydroLight output radiance,  $r_{rs}$  is calculated with (2,14)

$$r_{rs} = \frac{L_u(0^-)}{E_d(0^-)}$$
(8)

#### b. Monte Carlo Simulations

Since HydroLight is *not* commonly used to simulate waters of such high turbidity, a threedimensional forward Monte Carlo code (6) was developed to validate the results for those cases. The Monte Carlo code was run to compute  $r_{rs}$  with the same input parameters as HydroLight (Table 1). Upon reaching the water surface, the photons were consolidated by 240 solid angle 'quads', exactly as defined in the HydroLight program (10), so that the results can be compared. The incident radiation field was defined by using the subsurface downwelling radiance,  $L_d(0^-)$  from the output of a sample HydroLight run. For each set of IOP combination, approximately 10<sup>6</sup> photons were initiated per wavelength, with the number of photons per direction scaled by the normalised  $L_d(0^-)$  field.

In the Monte Carlo program, the  $r_{rs}$  is computed by (6)

$$r_{rs} = \frac{1}{N \cdot \Omega} \sum_{i=1}^{n} \frac{1}{|\cos \theta_i|}$$
(9)

where N is the number of downwelling photons,  $\Omega$  is the solid angle of the quad containing the exit direction, n is the number of upwelling photons reaching the surface, and  $\theta_i$  is the exit zenith angle of the *i*-th photon reaching the surface.

#### 4. Results

The  $r_{rs}$  computed from HydroLight is plotted against u in Figure 1, together with the Gordon88 and Lee04 models. The Lee04 model is not dependent on a single variable u, so we plot only the particle contribution against  $u_p$ . The water contribution in Lee04 is negligible in our region of interest (turbid waters).

In Figure 1, the  $r_{rs}$  from HydroLight simulations increases faster with u when u is large compared with the Gordon88 and Lee04 models. It diverges from Gordon88 at  $u \approx 0.3$ , and from Lee04 at  $u \approx 0.7$ . The increasing trend is clearly greater then linear, possibly due to the multiple scattering of photons on the suspended particles (15). From Eq. 3, the Lee04 model behaves linearly at high u, and is unable to account for the increased  $r_{rs}$ . While Gordon88 is a quadratic function of u, its increase is not fast enough to match the increase in  $r_{rs}$  at high u.

To make a meaningful quantitative comparison between the HydroLight data and the models, we divide u into 3 ranges of low, mid, and high u. The boundaries of these ranges are u = 0.4 and u = 0.8, near the points where the two models diverge significantly from the HydroLight results. The average percentage difference (APD) is then calculated<sup>1</sup>. As expected, both models do well in low u (*APD* < 10%). In mid u, Gordon88 has an APD of 16.9%, and Lee04, 2.6%. In high u, Gordon88 has an APD of 23.1%, and Lee04, 13.8%.

The APD at saturation (u > 0.95) was computed to show the maximum error at the highest turbidity levels. The maximum APDs were found to be 30.2% and 24.1% for Gordon88 and Lee04 respectively. The APDs between HydroLight and each of the models are summarised in Table 2.

To validate the HydroLight simulation results at high u, Monte Carlo simulations were run with similarly randomised IOPs. The Monte Carlo results for nadir-viewing  $r_{rs}$  is plotted in Figure 1. The relationship between  $r_{rs}$  and u from the Monte Carlo simulations follow a similar trend, resembling that of HydroLight. Like the previous observation, the rate of increase of  $r_{rs}$  is larger than both established models at high u.

<sup>&</sup>lt;sup>1</sup> For Lee04, the APD was calculated with the full model, *i.e.* with both water and particle terms.

## 5. Modelling $r_{rs}$ in High Turbidity

Results from both HydroLight and Monte Carlo suggest that the  $r_{rs}$  dependence on u is greater than the 2<sup>nd</sup> power. For the sake of reproducibility, we used the HydroLight results to develop the subsequent model. By doing a rough fit of the HydroLight data to various curves, we found that polynomials of 4<sup>th</sup>, 6<sup>th</sup>, and 8<sup>th</sup> order fit the points well. The 4<sup>th</sup> order polynomial fits most of the points up to  $u \approx 0.95$  where it diverges from the trend, while the 6<sup>th</sup> order polynomial fits the points at high u better.

The APD of the three polynomials with HydroLight are calculated and presented in Table 2. While the lowest APDs lie in the 6<sup>th</sup> and 8<sup>th</sup> order polynomial fitting, the APDs of the 4<sup>th</sup> order polynomial are already smaller than the uncertainty associated with  $r_{rs}$  or IOP measurements. As such, we choose to use the 4<sup>th</sup> order polynomial to model  $r_{rs}$  at high u.

#### 6. Development of a General Model

A previous model by Park & Ruddick (14) expressed the above-water remote sensing reflectance  $(R_{rs})$  as a 4<sup>th</sup> order polynomial of u. However, their model coefficients were a function of  $b_{bp}/b_b$  in addition to the sun-sensor geometry, to account for differences in the rate of increase of  $R_{rs}$  with u. Their HydroLight simulations of water types extended only up to u = 0.5. For practical usage, we aim to develop a model which is applicable across the whole range of u, with coefficients depending only on the sun-sensor geometry.

In the development of our general model that extends to high turbidity, a main consideration was to be able to reproduce the results of existing established models at low turbidity. In particular, the Lee04 model accounted for multiple g values (i.e. the ratio  $r_{rs}/u$ ) from the same u due to the different shapes of the water molecule and particle volume scattering functions (7). This effect is more significant at low u. A close fit is especially important in that range, since even a small deviation in  $r_{rs}$  would result in large percentage errors.

The HydroLight results (1,167,296 points per sun-sensor geometry set) were fitted to the following model

$$r_{rs} = g_w u_w + \sum_{i=1}^4 g_i u_p^i \,, \tag{10}$$

where  $g_w$ ,  $g_i$  are the model parameters dependent on the sun-sensor geometry.

To ensure that the error at low u is not amplified by the imperfect fit of the quartic polynomial, the HydroLight generated  $r_{rs}$  was initially fitted to the model in the low u region to determine values of  $g_w$ . Fixing  $g_w$ , a second fitting of HydroLight  $r_{rs}$  to the model was performed for all data points to obtain the coefficients  $g_i$  (i = 1,2,3,4) by minimising the absolute difference weighted by  $(1 - u_p)^2$ . The following relation was obtained for nadir-viewing sensors

$$r_{rs} = 0.099 \, u_w + 0.072 \, u_p + 0.296 \, u_p^2 - 0.363 \, u_p^3 + 0.240 \, u_p^4. \tag{11}$$

The model is plotted against u in Figure 2 with the HydroLight results, and the resulting APDs are listed in Table 2. The  $r_{rs}$  of HydroLight simulations versus the new quartic model is presented in Figure 3. As intended, the APDs are small from the low to high u range, and does

not exceed 5% in the saturation range. Examining the errors of all the points from Eq. 11, we found an average error of 0.3% and a maximum error of 7.6%.

In Table 3, we have listed the coefficients for commonly used sensor geometries (2,11,12).

# 7. Conclusion

From the results of the HydroLight and Monte Carlo simulations, it is determined that  $r_{rs}$  has at least a quartic dependence on u, possibly due to multiple scattering by particles. The HydroLight data points were fit to a quartic polynomial model, which included a term for water contribution based on the Lee04 model. From the fitting to this new quartic model, an average error of 0.3% and maximum error of 7.6% was found. This model is thus better suited to waters of high turbidity. In the event that higher accuracy in the saturation range is required, new coefficients can be derived from HydroLight simulations in that range of u, or by a different choice of weight in fitting HydroLight data to Eq. 10.

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# **Tables and Figures**

HydroLight/Monte Carlo	Input/Values		
Parameter			
Solar zenith angle	30°		
Wavelength (nm)	400 – 700, every 5 <i>nm</i>		
$a_g(440)$	Randomised between $0.03 - 2 m^{-1}$		
S	Randomised between $0.01 - 0.02$		
$b_{bp}(550)$	Randomised between $0.03 - 1 m^{-1}$		
γ	Randomised between $0 - 1.2$		
Particle scattering phase function	Fournier-Forand, $b_{bp}/b_p = 0.018$		
Water depth	Infinitely Deep		

Table 1: Input parameters used in HydroLight and Monte Carlo simulations.

Table 2: Average percentage difference between HydroLight results, and the specified model.

Model	Low <i>u</i>	Mid u	High <i>u</i>	Saturation	
	$0.0 < u \leq 0.4$	$0.4 < u \leq 0.8$	0.8 < u < 1.0	0.95 < u < 1.0	
Gordon88	9.4	16.9	23.1	30.2	
Lee04	2.3	2.6	13.8	24.1	
4 <sup>th</sup> Deg Poly	2.7	0.4	0.8	1.6	
6 <sup>th</sup> Deg Poly	1.2	0.3	0.4	0.6	
8 <sup>th</sup> Deg Poly	0.7	1.1	2.8	4.1	
New Quartic	0.5	0.2	0.7	4.7	

Table 3: Model coefficients for the new quartic model, with  $\theta_{sun} = 30^{\circ}$ .

Sensor Geometry	$g_w$	$\boldsymbol{g}_1$	$\boldsymbol{g}_2$	$\boldsymbol{g}_3$	${oldsymbol{g}}_4$
Nadir	0.099	0.073	0.296	-0.363	0.240
$ heta=20^\circ$ , $\phi=90^\circ$	0.100	0.074	0.304	-0.382	0.250
$ heta=40^\circ$ , $\phi=90^\circ$	0.103	0.079	0.319	-0.424	0.272
$\theta = 40^\circ$ , $\phi = 135^\circ$	0.092	0.082	0.335	-0.461	0.294



Figure 1: Scatter plot of  $r_{rs}$  on u from the HydroLight and Monte Carlo simulations. For comparison, the Gordon88 model is plotted, as well as the particle term from Lee04. The  $r_{rs}$  is taken at nadir.



Figure 2: Scatter plot of  $r_{rs}$  on u, of the new quartic polynomial model (on HydroLight u) and HydroLight results.



Figure 3:  $r_{rs}$  values of the new quartic model compared with  $r_{rs}$  values from HydroLight.